Calculation of Corona $V-I$
Characteristics of Monopolar Bundles
using the Charge Simulation Method

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ABSTRACT
In this paper, the charge simulation method is applied for modeling the $V-I$ characteristics of corona from monopolar bundled transmission lines without resort to Deutsch's assumption. The calculated results are compared with those obtained using Deutsch's assumption. The accuracy of the proposed method is discussed in the light of predicting values closer to those measured experimentally.

INTRODUCTION

HV dc has many advantages over conventional ac lines for long-distance power transmission [1]. Recently, the economic feasibility of dc transmission has increased with the development of HV terminal equipment. One of the problems associated with the HVDC transmission is the corona occurring on the transmission lines and the power loss, radio interference, television interference, and environmental impact associated with it.

Theoretical analysis of the ionized field around the transmission line conductors is very useful for design engineers to predict the corona power loss on the lines. In fact, the exact solution of the ionized field problem is extremely difficult due to the nonlinear nature of the governing equations. All attempts in the literature were based on some simplifying assumptions [2-4], among them Deutsch's and Kaptsov's assumptions.

Deutsch's assumption [5], stating that the presence of space charges has no effect on the direction of the resultant electric field, has been the basis of all work until that of Abdel-Salam and Khalifa [6,7]. These researchers were the first to break away truly from this persisting tradition which may account for the significant success of their calculation. Without resort to Deutsch's assumption, the resultant local field was computed in agreement with the physical understanding of the problem, as a superposition of the effects of system geometry and of space charge [6].

Kaptsov's assumptions [8] state that, after the initiation of corona the electric field intensity at the surface of the coronating conductor remains fixed in magnitude at the onset value in air. Meanwhile, experimental measurements by Waters et al. [9] and theoretical calculations by Khalifa and Abdel-Salam [10] have shown that this is not exactly true.

The finite-element method (FEM) has been applied by many investigators for calculating the space-charge-free field, e.g. [11,12]. Janischewskyj and Gela [13] have, for the first time, applied FEM to solve the ionized field in a simple coaxial cylindrical configuration, thus waiving Deutsch's assumption from the calculations, but retaining Kaptsov's assumption. Takuma et al. [14] succeeded in applying FEM to solve the ionized-field problem in the case of wire-plane geometry, using additional assumptions. They replaced Kaptsov's assumption with that of constant ion density: namely, the space charge density is assumed constant around the conductor periphery. Such an assumption is not realistic, especially for the case of bundle conductors where the charge density showed a significant change around the periphery of the subconductors [15]. Recently, Abdel-Salam...
The charge simulation method (CSM) has been applied by many investigators for calculating the space-charge-free field, e.g. [17,18]. Horenstein [19] has for the first time, applied CSM to approximate the electric field and space charge around a single conductor in corona and to compute the $V-I$ relationship for the discharge. More recently, Elmoursi and Castle extended the application of CSM to model the corona characteristics for a cylindrical precipitator [20] and later on for a wire-duct precipitator [21]. Very recently, the analysis of the ionized field was attempted using the FEM or CSM combined with the method of characteristics or the method of residues [22].

The most attractive property of CSM is that it is very accurate due to the smooth simulation of the surface of the coronating conductor with an equipotential surface.
On the other hand, approximation of the conductor surface with straight line segments, as adopted in FEM, results in a considerable error, especially in the immediate vicinity of the conductor. Also, CSM does not need iteration, as needed for FEM, with a subsequent saving of computational time. Therefore, the present authors prefer to extend CSM rather than FEM for their analysis of the ionized field around monopolar bundled transmission lines without resort to Deutsch's assumption.

First of all, the physics of monopolar corona is briefly reviewed and the equations describing the monopolar corona are presented with a formulation of the boundary conditions pertinent to their solution. Then, the method of analysis is explained showing how CSM can be applied for modeling the \( V-I \) characteristics of corona from monopolar bundled transmission lines without resort to Deutsch's assumption. Finally, the computed results are discussed in comparison with those measured experimentally and those calculated using Deutsch's assumption.

**PHYSICS OF MONOPOLAR CORONA**

Corona discharge is a self-sustained partial breakdown of air in the very thin 'ionization-layer' around the coronating subconductors of a given HVDC bundled transmission line. The electric field is so high inside the ionization-layer that the coefficient of ionization by electron collision exceeds the coefficient of electron attachment. For practical lines, the thickness of the ionization-layer is negligibly small in comparison with the height of line subconductors above the ground plane. The ionization-layer is considered to emit space charges (ions), which drift into the space between the subconductors and the ground plane. Eventually, the entire inter-electrode spacing is filled with monopolar space charges having the same polarity as the coronating subconductors.

The ions drift in the interelectrode spacing with a mobility which changes with their lifetime [23]. In this anal-
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The main system of equations describing the monopolar corona discharge is

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot \vec{J} = 0 \quad \text{or} \quad \int \vec{J} \cdot d\vec{A} = I \]

\[ \vec{J} = K \rho \vec{E} \]

\[ \vec{E} = -\nabla \phi \]

Equation (1) may be replaced by a single third-order non-linear partial differential equation:

\[ \nabla \cdot (\nabla \phi \nabla \phi) = 0 \] (2)

Equation (2) is the general form for the monopolar corona equation and there is no method known for solving it in the general case without resorting to some simplifying assumptions [2-5].

**BOUNDARY CONDITIONS**

The boundary conditions pertinent to the solution of the monopolar corona Equation (2) are given as follows

(1) The potential \( \phi \) at the coronating subconductors is equal to the applied voltage, i.e.

\[ \phi = V \] (3)
This means that the subconductor as a whole does not start corona at one voltage value, a phenomenon which has been observed experimentally [24].

The corona onset voltage is calculated based on the physical processes involved in corona discharge phenomena [25]. The onset voltage $V_o$ of the bundle is that corresponding to the point where the applied field is highest in its vicinity.

**METHOD OF ANALYSIS**

It is quite interesting to note that all methods reported so far in the literature [2-8] determine the corona current $I$ as a function of the applied voltage $V$, except for [19] which evaluates $V$ as a function of $I$. The present method of analysis seeks a determination of the applied voltage $V$ corresponding to a given value of the corona current $I.$
For the current $I$ to flow through the HV bundle, the applied voltage has to exceed the onset voltage $V_0$ by $\Delta V$:

$$V = V_0 + \Delta V$$

(5)

where $\Delta V$ is the required overvoltage above the onset level to establish space charges equivalent to the flow of the given current $I$. Therefore, the discrete infinite line charges used to simulate the bundled conductor-to-plane geometry are classified into two sets.

The first set, which will be referred to as the onset charges, correspond to the component $V_0$ of (Equation 5) and simulate the surface charge on the bundle at $V_0$. As explained before [17,18], the surface charge on each subconductor is represented by a number $n$ of unknown onset line charges uniformly distributed around a fictitious cylinder of radius $R$ inside the subconductor as shown in Figure 1(a), (b) for bundle 2. $R_I$ can take any value between $0.1R_s$ and $0.5R_s$ for acceptable accuracy of the solution, where $R_s$ is the subconductor radius. For a given bundle of $m$ subconductors, the total
The second set of simulation charges corresponds to the overvoltage component \( \Delta V \) of \( V \) (Equation (5)) and simulates the space charges filling the interelectrode spacing at the current value \( I \). To evaluate and locate these space charges, being represented by unknown line charges, a group of equipotential contours is found by extending a number \( M \) of radial trajectories starting from the bundle surface as shown in Figure 1c. The first equipotential contour was located by using only the onset charges, already determined, and calculating a voltage drop, e.g. \( \Delta \psi = 0.05 V_0 \), along the trajectories starting from points \((a, b, \ldots)\) on the bundle and terminating at points \((a', b', \ldots)\), Figure 1c. The space line charges of the first contour \( q_i \), \( i = 1, 2, \ldots M \) are located at the points \((a', b', \ldots)\). The magnitudes of the unknown charges \( q_i \) are determined such that they and their images maintain the contour equipotential at \( \psi_i \). Therefore, \( M \) boundary points \( p_j \), \( j = 1, 2, \ldots M \) are chosen.

However, the symmetry of subconductors about the vertical Y-axis, Figure 1, reduces the number of unknowns. To satisfy the potential Constraint (4) in the absence of space charge, images of all line charges with respect to the ground plane are considered, Figure 1. To determine the magnitudes of the unknown charges, \( n \) boundary points are chosen on the surface of each subconductor and a set of \( N \) equations relating the potential at these boundary points to the unknown charges is formulated [17,18],

\[
[P]_{N,N} \cdot [Q] = [V_0]_N
\]

where \([Q]\) is the magnitudes of unknown line charges, \([P]_{N,N}\) the associated potential-coefficient, and \([V_0]_N\) potential of the boundary points which is equal to \( V_0 \) for all the points. Using Gauss elimination techniques, the matrix Equation (6) is solved for the unknown onset line charges.

The number of unknown line charges is \( N = nm \).

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midway between the points \( (a', b', \ldots) \) and the calculated potential at each point is equated to \( \psi_1 \).

\[
\sum_{i=1}^{M} \frac{q_i}{2\pi\varepsilon_0} \ln \frac{d'_{ij}}{d_{ij}} = \psi_1
\]  

(7)

where \( d_{ij} \) and \( d'_{ij} \) are the distances measured from the \( i \)th line charge and its image to the \( j \)th boundary point. \( \varepsilon_0 \) is the permittivity of free space.

Application of Equation (7) at each boundary point formulates \( M \) equations into \( M + 1 \) unknowns (\( M \) unknown space line charges and the unknown potential value \( \psi \) of the contour).

The magnitudes of the unknown space line charges \( q_i \) must also satisfy the continuity of the current \( I \)

\[
\sum_{i=1}^{M} \left[ \frac{q_i}{\Delta r_i \Delta S_i} \right] K E \Delta S_i = I
\]  

(8)

where \( \Delta r_i \) is the radial distance between the bundle subconductors and the contour (or generally the distance between two successive equipotential contours) and \( \Delta S_i \) is the chord length between adjacent charges on the same contour. The quantity \( q_i/\Delta r_i \Delta S_i \) is the equivalent local space charge density \( \rho \). The electric field magnitude \( E \) is evaluated at the location of the \( i \)th charge in the direction normal to \( \Delta S_i \). \( K \) is the mobility. \( I \) is the corona current per unit length of the bundle and for which the applied voltage \( V \) (Equation (5)) is to be determined.

Equations (7) and (8) represent \( M + 1 \) equations in \( M + 1 \) unknowns whose solution yields the unknown \( M \) space line charges of the first contour.

As the contour is kept an equipotential, the contribution of the line charges \( q_i \) to the field inside the contour is zero with no influence on the onset voltage and its variation around the periphery of subconductors.

After the line charges of the first equipotential contour and their images are determined, their voltage and field contributions are included with the onset charges in integrating out towards the next equipotential contour and so on for each subsequent contour until the entire interelectrode spacing is filled with space charge. If the calculations have been properly performed, the integrated trajectories will eventually terminate on the ground plane.

The overvoltage \( \Delta V \) above the onset value \( V_o \) necessary to sustain the given current \( I \) is equal to the sum of the potential contributions of the space line charges on all the contours filling the interelectrode spacing and their images with respect to the ground, calculated at the subconductors surface. Then, the applied voltage \( V \) corresponding to the current \( I \) is \( V_o + \Delta V \) as dictated by Equation (5).

**NUMERICAL DATA**

In the present calculations, the number of onset charges \( n \) per subconductor is chosen to yield an equipotential surface which almost coincides with the subconductor surface. Twelve line charges per subconductor were found satisfactory as the calculated potential at the subconductor surface does not exceed 0.01% of the applied voltage.

Many attempts were made with varying the number of equipotential contours as well as the number of line charges per contour. Twenty contours, each with twelve line charges uniformly distributed around the contour, resulted in a proper termination of the integrated trajectories at the ground plane.

The mobility of ions \( k \) is considered constant at \( 1.5 \times 10^{-4} \) m\(^2\)/Vs for positive ions and \( 1.8 \times 10^{-4} \) m\(^2\)/Vs for negative ions.

**RESULTS AND DISCUSSION**

**EQUIPOTENTIAL CONTOURS AND FLUX LINES OF THE IONIZED FIELD**

Figures 2–4 show the equipotential contours around a conductor of radius 0.0102 m with a height of 9.35 m above the ground plane for different corona currents. Also some flux lines are traced in Figures 2–4.

It is quite clear that the flux lines of the ionized field deviate from those of the space-charge-free field, and the deviation increases with the value of the corona current. This deviation seems to contract the flux lines as a result of space charges. Such phenomena were interpreted and thoroughly investigated for the single conductor-plane geometry [6].
CORONA V-I CHARACTERISTICS

Before applying the proposed method for bundle conductors, it is worthwhile to check its correctness for predicting the V-I characteristic of a single conductor-plane geometry. Therefore, the calculated V-I characteristics are compared with the available experimental measurements.

For completeness, the calculated V-I characteristics are also compared with those obtained using the iterative method [4].

Figures 5-8 show the V-I characteristics calculated by the present method of analysis and by the iterative method [4] which resorts to Deutsch's assumption. The measured V-I characteristics are also plotted in Figures 5-8. Figure 9 shows the calculated V-I characteristic in relation to that measured experimentally.

Practical lines are normally of stranded construction and often contain surface irregularities. In the theoretical calculation of V-I characteristics, conductor surface irregularities may be included by substituting the actual conductor by an equivalent smooth conductor of the same diameter with a corona onset voltage reduced by a factor of 0.25 to 0.8, depending on the surface condition of the line conductors.

Figures 10 and 11 show the V-I characteristics of some practical lines predicted by the present method of analysis and by the experiment. Figures 11 and 12 show the V-I characteristics obtained by the iterative method in relation to that predicted by the present method.

It is quite clear that the present calculations are closer to the experimental values in comparison with those obtained by the iterative method. This is attributed to the effect of space charge on contraction of the field lines of the ionized field in comparison with those for the space-charge-free field. Such effect was disregarded by the iterative methods [2-4] as they are based on Deutsch's assumption.

Figures 13-15 show respectively the V-I characteristics of practical lines with bundle-2 horizontal, bundle-2 vertical, and bundle-4 subconductor arrangements. For comparison purpose, the V-I characteristic obtained by the iterative method [4] is also plotted.

Table 1. Error in Calculated Conductor Potential. $V(\psi)$ is the potential calculated at a point on the conductor surface at an angle $\psi$, measured clockwise from the vertical passing through the conductor axis.

<table>
<thead>
<tr>
<th>$\psi$ (°)</th>
<th>$I=0.5$</th>
<th>$I=1$</th>
<th>$I=3\mu A/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.0026</td>
<td>0.0041</td>
<td>0.0060</td>
</tr>
<tr>
<td>45</td>
<td>0.00015</td>
<td>0.00075</td>
<td>0.00277</td>
</tr>
<tr>
<td>105</td>
<td>-0.00019</td>
<td>-0.00059</td>
<td>-0.00189</td>
</tr>
<tr>
<td>132</td>
<td>-0.00186</td>
<td>-0.00249</td>
<td>-0.0042</td>
</tr>
<tr>
<td>165</td>
<td>-0.00108</td>
<td>-0.00189</td>
<td>-0.0043</td>
</tr>
</tbody>
</table>

Table 1 shows that the calculated voltage $V$ is almost...
the same around the periphery of the coronating conductor, irrespective of the corona current. This constancy of $V$ is also a measure of the accuracy of the proposed method of calculation.

Table 2 gives the computer time required by the present method for the calculation procedure at different values of the corona current i.e. the computer time required for calculating the applied voltage corresponding to these currents. The same table gives also the computer time required by the iterative method \cite{4} for calculating the corona current corresponding to the same set of applied voltage values. It is quite clear that the computer time of the present method is several times that required by the iterative method. It is the expense for waiving the calculation procedure from Deutsch's assumption. The computer time required by the present method depends, of course, on the number of equipotential contours chosen in the calculation procedure. The larger the number of contours, the longer is the computer time and the higher is the accuracy of calculation.

### CONCLUSIONS

1. The charge simulation method is applied for the analysis of a monopolar ionized field without resort to Deutsch assumption. This differs from the iterative method \cite{2-4} which is based on Deutsch’s assumption.

2. The proposed method of analysis predicts $V-I$ characteristics of corona from conductor bundles, irrespective of the number of subconductors per bundle.

3. Compared with the iterative method, the proposed method predicts $V-I$ characteristics closer to those measured experimentally.

4. The accuracy of the proposed method is at the expense of the computer time required for the calculation procedure, being longer than that required by the iterative method.

### REFERENCES

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