Image Enhancement using E-spline Functions

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Abstract- Exponential spline polynomials (E-splines) represent the best smooth transition between continuous and discrete domains. As they are constructed from convolution of exponential segments, there are many degrees of freedom to optimally choose the most convenient E-spline, suitable for a specific application. In this paper, the parameters of these E-splines were optimally chosen, to enhance the performance of image de-noising as well as image zooming schemes. The proposed technique is based on minimizing the total variation function of the detail coefficients of the E-spline based wavelet decomposition. In image de-noising schemes, apart from E-spline parameter estimations, the thresholding levels of their detail coefficients, are also optimally chosen. In zooming applications, the quality of interpolated images are further improved and sharpened by applying ICA technique to them, in order to remove any dependency. Illustrative examples are given to verify image enhancement of the proposed e-spline scheme, when compared with the existing approaches.

Keywords- Image de-noising, interpolators, E-spline functions

1. INTRODUCTION

During the past decade, there have been an increasing number of papers devoted to the use of polynomial splines in different signal processing applications, [1-3]. B-spline polynomials, is a class of these polynomial splines that find extensive applications in many engineering applications. In [4], a complete analysis for a B-spline Perfect Reconstruction (PR) frame work with a derivation for the scaling and wavelet functions was presented. However, as they are constructed using Haar functions, there is no much degree of freedom to use in optimizing the performance of some signal processing applications like the design of digital interpolators. On the other hand, Exponential splines enjoy a unique feature of being able to convert from analog to digital applications. This is crucial in several signal processing applications such as differential operators, fractional delays, interpolators and sampling rate converters, [5-6]. Moreover, E-splines has many degrees of freedom if they are optimized in a specific application, as they are constructed from the convolution of exponential segments with different rates.

In [7], a preliminary application for the usage of E-splines in image zooming and interpolation was presented. In this paper, it is proposed to use E-splines in enhancing the performance of image de-noising as well as image zooming schemes. In denoising applications, the proposed denoising technique is based on total variation function minimization, [8-9]. Using a recently developed E-spline wavelet decomposition, [10-14], the E-spline parameters as well as the thresholding levels of the E-spline detail coefficients are optimally chosen to minimize the total variation of the E-spline detail wavelet coefficients. In image zooming applications, E-spline based interpolators are used in image interpolation. In this case, the parameters of E-spline polynomials are chosen to boost the high frequency detail energy of the interpolated image. Further improvement is possible by estimating the missing high frequency details from the given low resolution image using the information contained in the whole low frequency image as described in sec. 4,[15-18]. Further image enhancement is also possible, by applying ICA techniques [19]; to the interpolated images in order to boost high frequency details and reduce any dependency between them. Illustrative examples are given to verify the ability of E-spline polynomials to significantly enhance image quality.

The paper is organized as follows: in sec. 2, a brief description of E-spline polynomials and wavelet Perfect Reconstruction PR systems is given. In sec. 3, the proposed denoising technique of E-spline polynomials is described. Sec. 4 describes the design of digital interpolator using E-spline polynomials. Sec. 5 concludes the paper.

2. MATHEMATICAL BACKGROUND

The Exponential \(m^{th}\) order spline polynomial \(B_{m}^{\alpha}(t)\), is constructed as \(m\) successive convolution of lower ones, i.e: \(B_{m}^{\alpha}(t) = B_{1}^{\alpha}(t) * B_{1}^{\alpha}(t) * ... * B_{1}^{\alpha}(t)\) (1)

where \(B_{1}^{\alpha}(t) = e^{\alpha t}, 0 \leq t \leq 1\). The vector \(\alpha\) can assume any positive, negative or even complex conjugate values. This means a considerable flexibility over Cardinal B-spline polynomials that only use Haar functions. \(B_{m}^{\alpha}(t)\) is of finite support and equals zeros at \(t \leq 0\) and \(t \geq m\). Between the knots \(t = 1,2,...,m-1\), it is represented by polynomials of order \((m-1)\) in \(t\), [5]. Due to its continuity and smoothness, it is used to expand continuous signals \(s(t)\). In the discrete case, \(s(n)\) can be expressed using the convolutional relation.

\[ s(n) = \sum_{k} c(k) B_{m}^{\alpha}(n-k) \]  

(2)

The \(c_k\) coefficients are obtained using the concept of
inverse filtering described in [4]. In [7], an alternate approach is given to determine these coefficients as the solution of the linear system

\[ s = BC, \]

\[ s = [s(1) \ s(2) \ldots s(N)]', \]

\[ c = [c(1) \ c(2) \ldots c(N)]' \]

\[ E_{1}(m) = \begin{bmatrix} B_{m}(m-1) & 0 & 0 \\ B_{m+1}(m) & B_{m+1}(m) & 0 \\ 0 & 0 & B_{m+2}(m) \end{bmatrix} \]

The solution of this system results in exact interpolation for any specified \( \delta \). The \( L \)-interpolated E-spline polynomial is defined by inserting \((L-1)\) equi-spaced points between every two knots of the E-spline polynomial, i.e. \( B_{m}^{(L)}(t) = B_{m}(t/L) \).

Quite recently [4], the complete perfect reconstruction E-splines wavelet family, has been constructed for any arbitrary choice of \( \alpha \). First, the E-spline 2-scale relation defined by

\[ B_{m}^{(2)}(t) = \sum_{k=-m}^{m} p(k) B_{m}^{(2)}(2t - k), \]

\[ P(z) = \frac{1}{2} \sum_{k=-m}^{m} p(k) z^{-k}, z = e^{j\alpha \pi} \]

has been determined for any arbitrary \( n \), \( \alpha \). Next, the wavelet E-spline function \( \psi_{m}^{(2)}(t) \), satisfying the orthogonality relation:

\[ \int_{-m}^{m} \psi_{m}^{(2)}(t) B_{m}^{(2)}(t-l) dt = 0 \]

i.e. \( \psi_{m}^{(2)}(t) = \sum_{k=-m}^{m} q(k) B_{m}^{(2)}(2t - k) \)

\[ Q(z) = \frac{1}{2} \sum_{k=-m}^{m} q(k) z^{-k} \]

Moreover, the Exponential dual scaling \( A(z) \) and wavelet E-spline functions \( B(z) \), have been constructed for any arbitrary \( \alpha \). Finally, it has been shown that, the following perfect construction (PR) relation is satisfied.

\[ P(z)A(z^{-1}) + Q(z)B(z^{-1}) = 1 \]

\[ P(-z)A(z^{-1}) + Q(-z)B(z^{-1}) = 0 \]

**Fig.1.** shows the complete PR E-spline wavelet system.

3. **E-SPLINE BASED IMAGE DENOISING**

In this section, we will show how to use E-spline wavelet family discussed above, in denoising noisy images, [8]. Wavelet decomposition amounts to representing signals by few nonzero coefficients. In case of a noisy data \( x \), that represents a signal \( f \) corrupted with uncorrelated zero-mean noise \( w \), i.e. \( x = f + w \), these coefficients are given by

\[ \langle x, \psi_{j,m} \rangle = \langle f, \psi_{j,m} \rangle + \langle w, \psi_{j,m} \rangle \]

where \( \langle \rangle \) is the inner product and \( \psi_{j,m} \) represents the \( j \)-scale wavelet basis used. As physical signals like speech and images, are nominally treated as baseband signals, the wavelet coefficients at fine scales, are mainly due to noise, and has to be thresholded. In [11], for signals corrupted with zero mean Gaussian noise, the threshold level is

\[ T = \sigma \log_{e}(2N) \]

\( N \) is the noisy signal length, while \( \sigma^{2} \) is the associated noise variance. The variance is estimated from the median \( M \) of the noisy signal, as \( \sigma = E(M)/0.6745 \). This technique is reported to give satisfactory results only for Gaussian noise. Recently [9-14], an efficient de-noising technique is performed by total variation minimization. The total variation function of a vector \( X \), is defined by

\[ TV = \sum |X(n+1) - X(n)| \]

For a matrix \( Y \), the total variation amounts to evaluating the derivatives along the horizontal and vertical axis. Denote these derivatives by \( Y_{x}^{(d)}(m,n), Y_{y}^{(d)}(m,n) \). The total variation along the \( x \) and \( y \) axis is defined as the \( \ell_{2} \) norm of \( Y_{x}^{(d)}(m,n), Y_{y}^{(d)}(m,n) \), respectively. Denote these norms by \( A_{x} \) & \( A_{y} \). In this paper, we take

\[ tv = \max(A_{x}, A_{y}) \]

So, it is proposed to choose \( \alpha \)'s of the E-spline function to minimize the total variation of the approximation sub bands while the threshold level \( T \), is chosen to threshold the E-spline detail coefficient such that the total variation of the complete; reconstructed denoised image \( \hat{Y} \) is minimum, the steps are summarized as follows:

1. For a prescribed decomposition level and initial choice of \( \alpha \)'s, construct E-spline wavelet decomposition of the noisy image.
2. Compute the total variation function of the approximation sub bands of the decomposed noisy image. Denote it by \( E_{1} \).
3. For any arbitrary choice \( T \), threshold the detail coefficient of the E-spline noisy image.
4. Reconstruct the image \( \hat{Y} \), using the E-spline synthesis filter banks. Evaluate the total variation function \( E_{2} \) of the reconstructed image.
5. Using any unconstrained optimization algorithm, find \( \alpha \)'s and \( T \), that minimize the following objective function

\[ \xi = ||Y - \hat{Y}||^{2}(1 - \beta_{1} - \beta_{2}) + \beta_{1}E_{1} + \beta_{2}E_{2} \]

where \( Y \) is the denoised image using the Median Based Variance estimation technique, [12-15], \( 0 < \beta_{1}, \beta_{2} < 1 \).
are (Lagrangian) multiplier. For simplicity we take \( \beta_1 = \beta_2 = 1/3 \).

The performance of the proposed E-spline de-noising, is compared with the Matlab Median based denoising technique as well as choosing the threshold level \( T \) only to denoise the total variation function of the detail coefficients of the noisy B-Spline decomposed image. 2-level wavelet decomposition was used. Figures (2-5) compare the performance of these cases, for Gaussian as well as Salt & Pepper noises, with different variances and concentrations.

These results verify the significant improved de-noising performance of the proposed E-spline technique. Note the poor performance of the Median based de-noising method for the Salt & Pepper case, as it was developed for the Gaussian noise case.

4. E-SPLINE SHARPENED INTERPOLATORS

To construct a super resolution image SR, from a given single low resolution image LR, [15-18]; one normally estimates the missing high frequency details from the given low resolution image using the information contained in the whole LR image. This is achieved as follows:

1. The LR image, \( (X_1) \), is decomposed into patches of \( N \times N \) non-overlapping blocks.
2. Each of these blocks scans the whole LR image to get the most \( M \) similar candidates to that block. Similarity is measured by those having the least distance in the first 4 moment.
3. Having obtained the \( M \) candidates, filter each of these candidates by a ‘Laplacian’ filter to estimate its high frequency energy. Denote these filtered components by \( X_i \), \( i=1...m \). To further estimate the high frequency content; decompose \( X_i \) by 1 level wavelet decomposition. Denote the energy in the detail bank of the decomposed \( X_i \) by \( E_i \).
4. An updated image with improved high frequency content is obtained as \( X_{up} = \sum(E_i / E) X_i \). These \( X_{up} \) images constitute an \( N \times N \) patches of the updated high frequency content of the LR image \( X_{1,LR} \).
5. For an arbitrary \( \alpha \) of the E-spline function, interpolate \( X_n = X_i + \mu X_{LR} \) to the desired interpolation level. The parameters \( \alpha \)'s and \( \mu \)'s are chosen to sharpen the interpolated image as follows:
   a) For an arbitrary \( \alpha \) and \( \mu \), interpolate \( X_n \) to the desired scale to yield an interpolated image \( Y_p \).
   b) Evaluate its high frequency energy as described in step 4 i.e.: simply by filtering the interpolated image by a ‘Laplacian’ filter. Denote its energy by \( E_d \).
   c) Using any unconstrained minimization technique, determine \( \alpha \) and \( \mu \) that minimize the following objective function: \( \| Y - Y_p \|^2 + 1/E_g \), where \( Y \) is the classical B-spline interpolated image of \( X_1 \).

The second step of sharpening the interpolated image \( Y_p \), is achieved by boosting the energy of its finest detail wavelet decomposition as follows:

1. Decompose \( Y_p \) by 1 level wavelet decomposition. \( 'bior4.4' \) wavelet family is used.
2. Filter its \( LH \) and \( HL \)-bands by an arbitrary \( MxM \) (e.g. \( M=3 \)), add the filtered output to the \( HH \)-sub-band. The parameters of this block are chosen to enhance the image through minimizing the total variation of the \( HH \) sub-band.
3. Using the updated detail bank, estimate the reconstructed image \( Y_r \).

Finally, in order to further boost high frequency detail energy and remove any dependence between the initial interpolated B-spline image \( Y \) and the optimized E-spline one \( Y_p \), these 2 images were processed by a Fast ICA algorithm FICA, [19]. The following table shows the final enhanced PSNR in dB of some standard Matlab images using regular B-splines, E-splines (1st step only without sharpening), E-splines (with sharpening) and E-splines (with sharpening and ICA boosting).

<table>
<thead>
<tr>
<th></th>
<th>Cameraman</th>
<th>Lena</th>
<th>Mandril</th>
<th>Boats</th>
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<tbody>
<tr>
<td>B-spl. Y</td>
<td>24.28</td>
<td>27.22</td>
<td>25.06</td>
<td>25.51</td>
</tr>
<tr>
<td>E-spl. Y_p</td>
<td>24.91</td>
<td>28.01</td>
<td>25.41</td>
<td>26.00</td>
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<tr>
<td>Sharp. E-spl Y_ab</td>
<td>25.17</td>
<td>28.44</td>
<td>25.64</td>
<td>26.12</td>
</tr>
<tr>
<td>ICA</td>
<td>25.20</td>
<td>28.33</td>
<td>25.60</td>
<td>26.15</td>
</tr>
</tbody>
</table>

As a final example, consider the 1074x810 Old man image as well as 274x184 Girl image. Both images were decimated using \( L = 2 \), to quarter of its size. Fig. (6-7), compare the performance of the classical B-spline based interpolators with the proposed sharpened super resolution ICA E-spline based interpolator. The resulting PSNR for Old man image for both interpolators are 32.67, 33.31 dB, respectively, whereas in the Girl image case, the resulting PSNR are 31.4604, 33.94 dB, respectively.

5. CONCLUSIONS

As a result of the extra degrees of freedom possessed by E-splines, they enjoy a higher level of energy concentration than their B-spline counterpart. Hence, when its parameters are correctly chosen, they can provide enhanced performance in different image de-noising as well as zooming applications. Simulation results have verified they improved denoising performance and its ability of constructing interpolators with an even sharper image than those available in the
existing methods. This is a crucial problem in SR system design. This work is funded in part by the Engineering center, Majmaah University, KSA.

6. REFRENCEES


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**Fig. 3:** Left: PSNR behavior of Exp-spline, B-spline and the Median based Matlab software `wdencmp`, for Salt & Pepper noise. Right: Noisy and de-noised images; PSNR in dB for noisy, Med. Based, B-spline and E-spline, are 18.94 20.85 22.79 23.09 dB, respectively.

**Fig. 4:** Left: PSNR behavior of Exp-spline, B-spline and the Median based Matlab software `wdencmp`, for Gaussian noise. Right: Noisy and de-noised images; PSNR in dB for noisy, Med. Based, B-spline and E-spline, are 15.78 23.31 23.3 23.708 dB, respectively.
**Fig. 5:** Left: PSNR behavior of Exp-spline, B-spline and the Median based Matlab software *wdencmp*, for Salt & Pepper noise. Right: Noisy and de-noised images; PSNR in dB for noisy, Med. Based, B-spline and E-spline, are 19.0, 21.7, 24.48, 24.54dB, respectively.

**Fig. 6:** Left: Decimated Old man image using L=2. Right: The performance of Old man image case using the classical B-spline and the proposed sharpened SR E-spline interpolation. The resulting PSNR are 32.67, 33.31 dB, respectively.

**Fig. 7:** Left: Decimated Girl image using L=2. Right: The performance of Girl image case using the classical B-spline and the proposed sharpened SR E-spline interpolation. The resulting PSNR are 31.4604, 33.94 dB, respectively.