Generating non-classical correlations between two superconductor qubits confined in a transmission cavity in dispersive limit under intrinsic noise

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ABSTRACT

An analytical solution of the dynamical evolution of two qubits confined in a transmission line cavity is obtained in dispersive limit under intrinsic noise. The dynamics of non-classical correlations are investigated by using different measures based on the skew information quantifiers [including local quantum uncertainty and uncertainty-induced non-locality], the Bell function and logarithmic negativity for experimentally chosen parameters. It is found that the generation of the non-classical correlations and its robustness depend crucially on the transverse exchange interaction, the dissipation rate, the mean photon number, and the frequency shift. The phenomena of sudden disappearance and sudden appearance of entanglement are observed. The stationary correlations appear due to the intrinsic decoherence, which depends on the parameters of the chosen initial states.

1. Introduction

Non-classical correlations (NCCs) between quantum objects are of fundamental and practical interests in quantum information [1,2], the most recognized type of the NCCs is quantum entanglement (QE) [3–5]. Recent studies show that the quantum information tasks can not be executed only by the QE [6], but there are other quantum correlations such as: quantum discord [6], and measurement induced non-locality (MIN) [7,8]. To avoid the difficulty of extracting analytical manipulations of geometric quantum discord (GQD), a variant is defined based on the p-norms as Hilbert–Schmidt norm [9] 2-norm, and Schatten 1-norm. However, the measurements that are based on 2-norm have been proved to be incompetent measures of NCC [10]. Consequently, the GQD and MIN are derived using 1-norm (trace norm) [11,12].

Other types of NCCs were introduced by using quantum skew information theory as local quantum uncertainty (LQU) [13], and uncertainty-induced nonlocality (UIN) [14]. LQU is considered as the new version of the MIN [14] in maintaining the fine computability but eliminating the non-well problem. In some general cases LQU was evaluated numerically [15–17], and it was analytically described for some special cases [18,19].

The interaction between matter, [especially two-levels atoms (qubits)], and the electromagnetic cavity field is a familiar physical concept to understand correlated states [20–24].

Thus, more applications appeared to understand the physics of quantum information. These applications for these qubit-field interactions, especially the artificial atoms, have attracted great interest because of their respective long decoherence time [25]. Among them the interaction of qubits with photons in a cavity which act as an interaction bus allowing a nonlocal coupling of the qubits. We can realize the concept of cavity bus as superconducting qubits placed face to face with all others in a superconducting transmission row resonator. The qubits are transmons (charge-phase qubit) [26], which are a moderated type of the Cooper pair box. In transmons, the qubits Josephson energy is greater than the charging energy and the jump frequency between the ground state and the first excited state depends on Josephson and charging energies [26].

Decoherence is the most fundamental problem in real-world applications of new technologies based on quantum processing [27,28]. In the dissipative systems, the useful NCCs are destroyed [29,30]. Various types of decoherence in real physical models are accountable for the quantum–classical switch. The intrinsic decoherence is one of them, which is defined as the automatically quantum coherence decay with the system evolution [31,32]. Both theoretical and experimental communities pay thus a lot of attention of combating decoherence in quantum computing architectures recently [33].

In this work we use Milburn equation for theoretical investigation of two qubits coupled to a cavity mode in the dispersive regime,
and a coherent, nonlocal coupling between the qubits. Milburn equation describes intrinsic decoherence models, which comes from the Schrödinger equation modified intrinsically, leading to decoherence [31].

Motivated by the above topics, LQO, UIN, and Bell correlations we investigate the quantum NCCs of charge-phase qubits in SC-cavity affected by the intrinsic noise, the photon numbers in the cavity, and Stark shift of the two qubits. The mutual interaction between the qubits is demonstrated clearly in terms of common states of the system [34]. After finding the analytical solution for the reduced density operator of the two qubits, we consider that the qubits are initially in superposition states. We believe that the advantage of this paper compared with the previous studies is the QCs computation for the density matrix of a general (non-X) two qubits state that can let us explore the effects of different parameters on the NCC generations. So we discuss the difference between the NCCs in terms of skew information, Bell-function and QE through the logarithmic negativity under intrinsic damping.

This article is organized as follows: Section 2 presents an analytical solution to the physical model. In Section 3, the measures of quantum correlations are summarized. Section 4 is about the computational results and their discussion. Finally, in Section 5 concluding remarks are shown.

2. The model and its analytical solution

The physical model includes two qubits confined in a transmission line cavity in the dispersive limit. The transmon charge-phase qubit is a modified version of the Cooper pair box, where the Josephson energy is greater than the charging energy and the jump frequency between the ground state and the first excited state depends on these energies. In the dispersive limit, no energy is exchanged with the transmission line cavity. The exchange interaction between the qubits is taken in consideration. The effective hamiltonian is given by [35]:

\[ \hat{H} = \frac{\hbar}{2}(\omega_1 \sigma_1^+ \sigma_2^+ + \omega_2 \sigma_2^0) + \hbar(\omega_1 \sigma_1^+ \sigma_1^0 + \omega_2 \sigma_2^0) \hat{a}^\dagger \hat{a} + \hbar J(\sigma_1^0 \sigma_2^+ + \sigma_1^+ \sigma_2^0), \] (1)

where \( \hat{a}^\dagger \) and \( \hat{a} \) are the operators of the transmission line cavity with frequency \( \omega_c \). The operators \( \sigma_i^+ \) and \( \sigma_i^0 \) are the Pauli matrices of the qubits with the frequencies \( \omega_i \). The \( \chi_i \) is the frequency shift of \( i \)-qubit which are equivalent the Stark shift of the qubit frequencies [36]. The last term describes the interaction of the qubits with each other, which is an exchange interaction between the qubits. Here, we focus on the intrinsic noise effects; therefore, the dynamics of the system, Milburn equation becomes [31]. For the density operator \( \rho \),

\[ \frac{d\hat{\rho}}{dt} = \hat{L}\hat{\rho}(t), \] (2)

where the super-operator \( \hat{L} \) is given by

\[ \hat{L}\hat{\rho} = -i [\hat{H}, \hat{\rho}] - \gamma(\hat{\rho}\hat{H} + \hat{H}\hat{\rho}), \] (3)

where \( \gamma \) is the intrinsic decoherence rate parameter.

To explore the non-classical correlations under the intrinsic noise, the solution of the entire system of Eq. (2) must be found for appropriate initial states. Therefore, we suppose the initial density matrix of the two qubits and the cavity is given by

\[ \rho(0) = \sum_{m,n=0,1} q_{mn}|m\rangle \langle n| \otimes \rho^{AB}(0). \] (4)

Where the cavity field is prepared initially in a coherent state \( |\psi_c\rangle = \sum q_{n} |n\rangle \) with \( q_{n} = e^{-\frac{n^2}{2}} \) and \( \bar{n} = |\psi_c|^2 \) is the mean photon number.

To investigate the dynamics of the generated NCCs amount due to the unitary interaction of the two qubits confined in a transmission cavity in dispersive limit under intrinsic noise, the initial state of the two qubits is prepared in a uncorrelated state as: \( |\psi_i\rangle = |\varphi_A\rangle \otimes |\varphi_B\rangle \), therefore, \( \rho^{AB}(0) = |\varphi_i\rangle \langle \varphi_i| \).

In the space states \(|11\rangle = |1_1, 1_2\rangle, |22\rangle = |1_1, 0_2\rangle, |33\rangle = |0_1, 1_2\rangle, |44\rangle = |0_1, 0_2\rangle \), the Hamiltonian dressed states \( |\Psi^n_{ij}\rangle = C_{ijk} |1\rangle + C_{ijk} |2\rangle + C_{ijk} |3\rangle + C_{ijk} |4\rangle \) \( \otimes \) \( |n\rangle \),

Solving the eigenvalue problem for the coefficients \( C_{ijk} \) we arrive to the following eigenstates:

\[ |\Psi^1_{11}\rangle = |1\rangle \otimes |n\rangle, \]
\[ |\Psi^2_{11}\rangle = |2\rangle \otimes |n\rangle, \]
\[ |\Psi^3_{11}\rangle = \frac{1}{\sqrt{J^2 + J^2}} [J\langle 1\rangle \otimes |n\rangle - J\langle 3\rangle \otimes |n\rangle], \]
\[ |\Psi^4_{11}\rangle = \frac{1}{\sqrt{J^2 + J^2}} [J\langle 2\rangle \otimes |n\rangle - J\langle 3\rangle \otimes |n\rangle]. \] (7)

where \( J_1 = \delta \pm \sqrt{\delta^2 + J^2} \), and \( \delta = (\chi_1 - \chi_2) n + (\omega_1 - \omega_2)/2 \). The dressed states \( |\Psi^n_{ij}\rangle \) are corresponding to the eigenvalues \( E^n_{ij} \) given by:

\[ E^n_1 = (\omega_1 + \chi)n + (\omega_1 + \omega_2)/2, \]
\[ E^n_2 = (\omega_1 - \chi)n - (\omega_1 + \omega_2)/2, \]
\[ E^n_3 = \omega_0 n + \sqrt{\delta^2 + J^2}, \]
\[ E^n_4 = \omega_0 n - \sqrt{\delta^2 + J^2}, \] (8)

where \( \chi = \chi_1 + \chi_2 \).

Using Eq. (3), the time evolution for the dressed states operators, \( \hat{X}^{\mu\nu}_{ij} = (|\Psi^{\mu\nu}_{ij}\rangle \langle \Psi^{\mu\nu}_{ij}|) \), are given by

\[ \dot{\hat{X}}^{\mu\nu}_{ij} = e^{-i(\epsilon_\mu E^n_{ij} + \epsilon_\nu E^n_{ij})}|\Psi^{\mu\nu}_{ij}\rangle \langle \Psi^{\mu\nu}_{ij}|, \] (9)

where \( e^{-i(\epsilon_\mu E^n_{ij} + \epsilon_\nu E^n_{ij})} \) is the intrinsic noise term. In the next section, we use the quantifier of both the skew information and Bell-equation, and the logarithmic negativity to investigate the generation of the NCCs between the two qubits. To do that, we look for the reduced density matrix of the qubits, \( \rho^{AB}(t) \) which we can obtain by tracing the transmission line cavity degree of freedom as follows:

\[ \dot{\hat{\rho}}^{AB}(t) = \sum_{k=0}^{\infty} \langle k| \dot{\hat{\rho}}(t) |k\rangle. \] (10)

From Eq. (4) and using Eqs. (9) and (10), the reduced density matrix of the qubits has finally the following expression:

\[ \hat{\rho}^{AB}(t) = \sum_{\mu=0}^{4} \sum_{\nu=0}^{4} \sum_{i,j} \hat{a}_{\mu\nu} \langle \Psi^{\mu\nu}_{ij}\rangle \langle \Psi^{\mu\nu}_{ij}|, \] (11)

where \( \rho_{ij} \) the elements of the reduced density matrix \( \hat{\rho}^{AB}(t) \), are given by:

\[ \rho_{11} = 1/4; \]
\[ \rho_{12} = 1/8\sqrt{\delta^2 + J^2}; \]
\[ \rho_{13} = 1/8\sqrt{\delta^2 + J^2}; \]
\[ \rho_{14} = 1/8\sqrt{\delta^2 + J^2}; \]
\[ \rho_{22} = 1/16(\delta^2 + J^2); \]
\[ \rho_{23} = 1/16(\delta^2 + J^2); \]
\[ \rho_{24} = 1/16(\delta^2 + J^2); \]
\[ \rho_{33} = 1/16(\delta^2 + J^2); \]
\[ \rho_{34} = 1/16(\delta^2 + J^2); \]
\[ \rho_{44} = 1/16(\delta^2 + J^2); \]
\[ \begin{align*}
\theta_{13} &= \frac{1}{16\delta^2}(2J\delta \beta_{13}^+ + \beta_{13}^- - 4J\delta + 4(\delta^2 + J^2)), \\
\theta_{14} &= \frac{1}{8\sqrt{\delta^2 + J^2}}((-J_x + J)\beta_{14}^+ + (J_x - J)\beta_{14}^-), \\
\theta_{44} &= \frac{1}{4},
\end{align*} \]

and \( \theta_{ij} = \theta_{ji}^* \) (\( i, j = 1, 2, 3, 4 \)), where \( \beta_{ij}^\pm = e^{\pm i(\xi_1^\pm - \xi_2^\pm + \gamma_1^\pm - \gamma_2^\pm)} \).

Once this analytical solution is found, then quantum phenomena; such as non-classical correlations in the considered physical model can be studied.

3. Measures of non-classical correlations

Here some of NCC measures of a bipartite state \( \rho_{AB}(t) \) are recalled based on the maximal violation of Bell inequality and skew information theory, including LQU, UIN [37], as well as the logarithmic negativity function.

3.1. Logarithmic negativity

The entanglement of a bipartite state of two-qubit described by \( \hat{\rho}_{AB}(t) \) is parametrized using logarithmic negativity. The logarithmic negativity is given by [38]:

\[ N(t) = \log(1 + \sum_{i,j=1}^{4} \xi_i^j), \]  

(13)

where \( \xi_i^j \) are the negative eigenvalues of \( \hat{\rho}_{AB}(t)^{T_A} \) that is the partial transpose of the density matrix \( \hat{\rho}_{AB}(t) \) with respect to qubit \( A \) of the two qubits. The elements of \( \hat{\rho}_{AB}(t)^{T_A} \) are given by

\[ \langle i, j | (\hat{\rho}_{AB}(t)^{T_A}) | m, n \rangle = \langle m, j | \hat{\rho}_{AB}(t) | i, n \rangle. \]

(14)

The logarithmic negativity takes values between 0 and 1, which are the minimum and maximum values corresponding to separated and maximally entangled states, respectively.

3.2. Bell-function

If a bipartite state \( \hat{\rho}_{AB} \) may be written in its Bloch representation as follows:

\[ \hat{\rho} = \frac{1}{4} I^A \otimes I^B + \frac{3}{4} \sum_{i=1}^{3} (x_i \sigma_i \otimes I^B + I^A \otimes y_i \sigma_i) + \sum_{i=1}^{3} t_{ij} \sigma_i \otimes \sigma_j, \]  

(15)

then correlation matrix of the state \( \hat{\rho}_{AB} \) is \( T = [t_{ij}] \), where the non-vanishing correlation matrix elements are given by [37]:

\[ t_{ij} = \text{Tr}(\hat{\rho}_{AB} \sigma_i \otimes \sigma_j)), \]

(16)

where \( x_i = \text{Tr}(\hat{\rho}_{AB} \sigma_I \otimes I)), y_i = \text{Tr}(\hat{\rho}_{AB} (I \otimes \sigma_j)) \) are the local Bloch vector \( \hat{x} \) and \( \hat{y} \) components, and \( | \sigma_i \rangle \) are the Pauli spin matrices. Where, it is known that \( \hat{\rho}_{AB} \) is non-degenerate if and only if \( \hat{x} \neq 0, \hat{y} \neq 0 \) [37].

An important quantifier based on the correlation matrix is that takes the maximal violation of Bell’s function [39]. If Bell-function is greater than the classical threshold 2, the Bell-violability is violated, it, i.e., it presents a nonlocal correlation. The analytical formula of the maximum Bell function for a two-qubit state, \( \rho_{AB} \), is given by

\[ B(t) = 2(\max_{i,j} (\lambda_i + \lambda_j)^{0.5}), \quad i, j = 1, 2, 3, \]

(17)

where \( \lambda_i \) are the eigenvalues of the matrix \( T^T T \).

3.3. Measures of skew information

Skew information (SI) is defined as the known information of a qubit that can be known by the other qubit. The amount of SI oscillates depending on the correlation amount between the two qubits and it vanishes when the considered state and the observable commute. The SI for a two-qubit state \( \hat{\rho}_{AB} \) is defined as

\[ I(\hat{\rho}_{AB}, K^A) = \frac{1}{2} Tr(\sqrt{\hat{\rho}_{AB}} K^A \hat{\rho}_{AB} K^A), \]

(18)

where \( K^A = K_A^A \otimes \Pi_B \) is a local observable, \( K_A^A \) is a Hermitian operator on subsystem \( A \) for a non-degenerate spectrum \( A \). The SI is dependent on the non-commutativity among \( \hat{\rho}_{AB} \) and \( K^A \), it can be used as an uncertainty measure of the observable \( K^A \) in the density matrix \( \hat{\rho}_{AB} \) state. Here we use two measures based on the SI namely: Local quantum uncertainty (LQU) and Uncertainty-induced non-locality measure (UIN) to investigate the NCCs between the two qubits considered in the article.

(1) LQU:

Local quantum uncertainty function may be considered as a well-defined quantum correlation measure [12] for bipartite quantum systems based on SI, which is defined as [13]:

\[ \hat{\mathcal{C}}_A(\hat{\rho}_{AB}) = \min_{K_A^A} I(\hat{\rho}_{AB}, K^A). \]

(19)

where the minimum runs over all the local maximally informative observables. The LQU estimates the minimum value of the uncertainty of \( K^A \) in \( \hat{\rho}_{AB} \) state. If LQU value is not equal to zero, the uncertainty of the observable in \( \hat{\rho}_{AB} \) appears, and it vanishes for all the states that have zero discord, so it is used to explore the discord-like quantum correlation. For a reduced density matrix for two-qubit system, \( \hat{\rho}_{AB} \), the closed form of LQU is given by

\[ \hat{\mathcal{C}}(t) = 1 - \lambda_{\text{max}}(W_{AB}), \]

(20)

where \( \lambda_{\text{max}} \) is the maximum eigenvalue of the \( 3 \times 3 \) matrix \( W_{AB} \), whose elements \( w_{ij} \) are calculated from:

\[ w_{ij} = Tr(\sqrt{\hat{\rho}_{AB}} (\sigma_i \otimes I_3) \sqrt{\hat{\rho}_{AB}} (\sigma_j \otimes I_3)). \]

(21)

(2) UIN:

For a bipartite state, \( \rho_{AB} \), the UIN is defined as [14]:

\[ U(\hat{\rho}) = \max_{K_A^A} I(\rho_{AB}, K^A). \]

(22)

UIN is regarded as the updated version of the known measurement-induced nonlocality (MIN). UIN quantifies the maximum value of SI between the given state \( \hat{\rho}_{AB} \) and the observable \( K^A \). UIN like LQU is also invariant under local unitary operation, and non-increasing under local operation on the subsystem \( B \), but they vanish for the product states. The closed form of UIN is given by [14]:

\[ U(t) = \begin{cases} 
1 - \frac{1}{\delta^2} \bar{x} W_{AB} \bar{x}^T, & \text{if } \bar{x} \neq 0; \\
1 - \lambda_{\text{min}}(W_{AB}), & \text{if } \bar{x} = 0.
\end{cases} \]

(23)

The minimum runs over all the matrix elements of \( W_{AB} \) given in Eq. (21).

4. Numerical results

We use Eq. (11) and analyze the generated NCCs between the two qubits, due to the unitary interaction under the dissipation rate of the intrinsic noise. We consider the effect of mean photon number, a.c. Stark shift values, and the transverse exchange interaction strength \( J \) between the qubits, on the \( \hat{\mathcal{C}}(t), U(t), B(t), \) and \( N(t) \) through the following figures.

In Figs. 1–4, we plot the time evolution of the NCC functions \( \hat{\mathcal{C}}(t), U(t), B(t), \) and \( N(t) \) with the experimental parameters of the qubits frequencies \( \omega_q \) and the resonant frequency \( \omega_r \) when the system...
starts with the superposition initial state given by Eq. (5) as $|\psi\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |10\rangle + |01\rangle + |00\rangle)$. Therefore, initially the NCCs do not exist; $\hat{\chi}(0) = \hat{U}(0) = N(0) = 0$, and violation of Bell inequality does not occur ($B(0) = 2$).

In Fig. 1(a), the generated NCCs have a quasi-periodic behavior when the mean photon number variation; $\bar{n} = 1$, the transverse exchange interaction constant $J = 0$, and the Stark shift values of $(\chi_1, \chi_2) = (3, 0)$ in the absence of the intrinsic noise effect, $\gamma = 0$.

Also one can see that the LQU and UIN present the same correlation that is called the skew-information correlation [17] (i.e., $\hat{\chi}(t) = \hat{U}(t)$). The effect of Stark shift on the generated NCC is the same of positive or negative signal of $\chi_j$. One can observe that the functions $\hat{\chi}(t)$ and $\hat{U}(t)$ reach their maximum values $\hat{\chi}_{\text{max}}(t) = \hat{U}_{\text{max}}(t) = 1$ during different intervals [which correspond to maximum steady correlation (MSC)].

After that they decrease partially to present partial correlations. The Bell function takes the values: $B(t) > 2$ in some time intervals with oscillatory behavior, i.e., the Bell function correlation arise due the unitary interaction of the two qubits with the transmission line cavity in the dispersive limit. The negativity $N(t)$ has oscillatory behavior and presents partial QE.

In Fig. 1(b) we investigate the dependence of the generated correlation on the mean photon number when increased to $\bar{n} = 4$, when the other parameters are fixed. By comparing Fig. 1(a), and 1(b), we find the following: (i) The phenomenon of entanglement sudden birth and sudden death [41,42] appear with increasing of the mean photon number. (ii) One can observe the elongation of the MSC intervals. (iii) The Bell correlation function, $B_{\text{max}}(t)$ lower bounds values are decreasing with increasing of the mean photon number $\bar{n}$, and the Bell’s inequality violation $B_{\text{max}}(t) > 2$ appears at shorter intervals.

Measuring the cavity phase shift enables us to simultaneously read-out the states of both qubits using a single line proportional to $X_1 \delta^2_1 + X_2 \delta^2_2$, whose effects are discussed in Fig. 2(a,b) with different Stark shift values $\chi_j$. From Eqs. (7),(8) we can see that the solution depends on $\chi_j$ in two ways: their sum in $\chi_j$ and difference in $\delta$; therefore we will compare Fig. 2 with Fig. 1(a) to enable us to show the effects of $\chi_j$, $\delta$. One can observe in Fig. 2(a),(b) that LQU and UIN have oscillatory behaviors in the maximum steady state intervals and behave symmetrically with the entanglement $N(t)$. Also, LQU exhibits quick changes more than UIN during its time evolution [43].

The value of $\delta$ in the case of Fig. 1(a) is greater than it in Fig. 2(a). Thus, we can observe that: The NCC oscillating frequencies increase with the increasing of $\delta$-value, and the interaction time intervals in which the Bell’s inequality is violated decrease with the decreasing of $\delta$. Thus decreasing these parameters makes the NCC more robust to the changes.

Fig. 2(b) has greater $\chi$-value than Fig. 1(a),(a); so we can see the increasing in the robust behavior of the Bell correlation function, $B_{\text{max}}(t)$ with high $\chi$-value. While the MSC intervals suffer from small fluctuation.

Fig. 3 shows the effect of the exchange interaction constant $J$. The correlations have more robust behavior in the same oscillating intervals with $J$ increasing. LQU and UIN become less stable in the MSC intervals, and give the same correlation (skew-information correlation) when transverse exchange interaction strength $J$ between the qubits increases. Also, there are small fluctuation in their intervals and they reach the maximum value 1.

Fig. 4 represents the NCC functions for different values of $J$, $\chi_1$, $\chi_2$, experimentally achieved [40] when $\gamma = 0$, and 0.005. The dissipation rate $\gamma$ appearance in Fig. 4(b), leads their amplitudes to a reduction and the entanglement to disappear. Due to the decoherence term: $e^{-\gamma t} \langle E_1^2 E_2^2 \rangle$ the generated correlations do not return to their initial values and positioned finally at their stationary steady states. The NCC functions have damped oscillatory behavior. After long time, the negativity vanishes due to the coupling with the environment, whereas
\(\hat{\chi}(t)\), and \(\hat{U}(t)\) get asymptotically the same non-zero stationary values (stationary skew-information correlation). It is worth mentioning that the skew information and \(1 - i\text{Tr}(\rho A^2)\) follow the same behavior, as predicted for the UIN [14]. Also, \(B_{\text{max}}(t)\) settles, but with the highest stationary value of the correlations.

5. Conclusion

The physical model of two SC-qubits confined in a transmission line cavity is analytically solved in dispersive limit under intrinsic noise. The effects of the intrinsic decoherence, the interaction strength between the two qubits, Stark shift of the qubits, and the initial mean photon number are investigated. The generated NCCs between the two qubits are analyzed by using different functions for the UIN, LQU, and the Bell function, as well as the logarithmic negativity. It is found that these functions show different oscillatory behavior except the UIN and LQU. In the case of equal UIN and LQU (in which the minimization of SI equals to its maximization), the NCC is called the skew-information correlation. The generated NCCs depend on the exchange interaction, the dissipation rate, the initial intensity coherence and the Stark shift. We can see that the NCC of UIN and LQU is more robust against decoherence rate than the Bell function and the logarithmic negativity. The appearance of the skew-information correlation depends on the variations of the Stark shift parameters. The intrinsic noise rate leads to keeping NCC functions at their non-zero stationary values, except the logarithmic negativity which vanishes. The amounts and oscillatory behavior of NCCs can be enhanced by parameters of the interaction strength between the two qubits, and the Stark shift of the qubits. Thus justifiable choices of the parameters of the Stark shift, exchange coupling, and mean photon number can result on extracting more information from the proposed system.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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