SHEAR BEHAVIOUR OF RC BEAMS STRENGTHENED EXTERNALLY WITH BONDED CFRP–U STRIPS

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This paper aims to contribute to a better understanding and modeling of the shear behaviour of RC beams strengthened externally with carbon fiber reinforced polymer CFRP–U strips. Nine RC beams without internal shear reinforcement were tested; one beam was kept as a control beam; whereas other beams were strengthened externally with CFRP–U strips. Test variables were, effective height (depth) of CFRP–U strips, number and width of strips for the same shear reinforcement ratio, and spacing or amount of strips. Test results showed that, the ultimate shear capacity of RC beams can be increased significantly using this techniques, a maximum increase of 93% was obtained. For beams strengthened with the same amount of CFRP strips, increasing the number of fiber by decreasing the strip width has a slight effect on both carrying capacity and ductility of the beams. A simple improved model has been proposed to predict the contribution of CFRP–U strips to the shear capacity of the strengthened beams, which fail by CFRP debonding. Research recommendations have been given at the end.

KEYWORDS: RC beams, shear strengthening, carbon fiber reinforced polymer CFRP–U strips.

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>Width of beam cross–section (web width), mm.</td>
</tr>
<tr>
<td>( d )</td>
<td>Effective depth of beam, mm</td>
</tr>
<tr>
<td>( E_f )</td>
<td>Modulus of elasticity of CFRP, MPa.</td>
</tr>
<tr>
<td>( f_{cu} )</td>
<td>Cube compressive strength of concrete, MPa.</td>
</tr>
<tr>
<td>( f_{fe} )</td>
<td>Effective stress of CFRP–U strips, intersected by shear crack, MPa.</td>
</tr>
<tr>
<td>( f_{fu} )</td>
<td>Ultimate tensile strength of CFRP–strips, MPa.</td>
</tr>
<tr>
<td>( h_{fe} )</td>
<td>Effective height or depth of CFRP–U strips, mm.</td>
</tr>
<tr>
<td>( K )</td>
<td>Bond length ratio.</td>
</tr>
<tr>
<td>( L_e )</td>
<td>Effective bond length, mm.</td>
</tr>
<tr>
<td>( N_f )</td>
<td>Number of CFRP–U strips per shear span.</td>
</tr>
<tr>
<td>( R )</td>
<td>Ratio of effective to ultimate strength of CFRP strips.</td>
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</tbody>
</table>
\[ S_f = \text{Central spacing of CFRP–U strips, measured along the long. axis of beam, mm.} \]
\[ t_f = \text{CFRP effective thickness, mm.} \]
\[ V_f = \text{Shear contribution of CFRP–U strips, to shear capacity of beam, kN.} \]
\[ V_{f,\text{exp.}} = \text{Experimental shear contribution of CFRP–U strips, kN.} \]
\[ V_{f,\text{pre.}} = \text{Predicted shear contribution of CFRP-U strips, kN.} \]
\[ V_u = \text{Ultimate shear capacity of strengthened beam, kN.} \]
\[ W_f = \text{Width of CFRP strips, mm.} \]
\[ \epsilon_{fe} = \text{Effective strain of CFRP–U strips, at failure.} \]
\[ \epsilon_{fu} = \text{Ultimate tensile (rupture) strain of CFRP strips.} \]
\[ \rho_f = \text{CFRP shear reinforcement ratio, } \left( \rho_f = \frac{2 t_f W_f}{b S_f} \right). \]
\[ \epsilon_{\text{max.}} = \text{Maximum tensile strain in CFRP-strips measured just before failure.} \]

1. INTRODUCTION

Some of the existing reinforced concrete structures may require strengthening or stiffening in order to increase their structural performance. Strengthening with adhesive bonded fiber reinforced polymer (FRP) has been established as an effective method applicable to many types of such structures.

Several studies have been focused on the potential use of FRP for flexural strengthening of concrete beams, but relatively little research has been done on the use of FRP in shear strengthening [1-12]. In addition to that the current understanding of the shear behaviour of RC beam strengthened with FRP is limited and much further research is still needed. Therefore, the aims of this study were to gain a better understanding and enhance the experimental database of shear behaviour of RC beams, without internal shear reinforcement, strengthened externally with bonded CFRP–U strips, and to develop a simple accurate model to predict the contribution of CFRP–U strips to the shear capacity of such beams at the complete debonding of the critical CFRP strips. The main variables investigated were, effective height (depth) of CFRP–U strips, number and width of strips for the same amount of fiber shear reinforcement, and spacing or amount of strips.

2. EXPERIMENTAL PROGRAM

2.1 Test Specimens

A total of nine 120 x 300 x 2150 mm concrete beams were tested in this study. All beams were designed to fail in shear mode rather than flexure, and no internal shear reinforcement was provided to assess the actual increase in shear strength by devised strengthening technique. One beam was kept as control.
beam, whereas other beams were strengthened in shear spans with CFRP–U strips. Table (1) and Fig. (1) give summary of testing program and specimens details. The parameters investigated in this study included, effective height of CFRP–U strips \((h_{fe})\), number \((N_f)\) and width \((W_f)\) of CFRP–strips for the same shear reinforcement ratio \((\rho_f)\), and spacing \((S_f)\) or amount of CFRP strips \((\rho_f)\).

Table 1: Specimens details

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>(f_{cu}) MPa</th>
<th>External fiber reinforcement (CFRP–U strips)</th>
<th>(W_f) (mm)</th>
<th>(S_f) (mm)</th>
<th>(h_{fe}) (mm)</th>
<th>(N_f)</th>
<th>(W_f/S_f)</th>
<th>(\rho_f) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B–1</td>
<td>33.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B–2</td>
<td>33.5</td>
<td>60</td>
<td>120</td>
<td>120</td>
<td>6</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–3</td>
<td>33.5</td>
<td>60</td>
<td>120</td>
<td>180</td>
<td>6</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–4</td>
<td>34.0</td>
<td>60</td>
<td>120</td>
<td>240</td>
<td>6</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–5</td>
<td>34.0</td>
<td>30</td>
<td>60</td>
<td>240</td>
<td>12</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–6</td>
<td>35.0</td>
<td>90</td>
<td>180</td>
<td>240</td>
<td>4</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–7</td>
<td>33.0</td>
<td>120</td>
<td>240</td>
<td>240</td>
<td>3</td>
<td>0.50</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>B–8</td>
<td>32.0</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>4</td>
<td>0.33</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>B–9</td>
<td>32.0</td>
<td>60</td>
<td>240</td>
<td>240</td>
<td>3</td>
<td>0.25</td>
<td>0.050</td>
<td></td>
</tr>
</tbody>
</table>

2.2 Materials

– Concrete mix design was made to produce normal strength concrete having a 28 day cubic compressive strength of 30 MPa. Ordinary Portland cement, local natural sand and gravel of 20 mm maximum size were used.
– Two diameters of high strength deformed bars 22 and 12 mm of 440 and 470 MPa proof strength respectively were used for longitudinal reinforcement.
– Uniaxial Carbon Fiber Reinforced Polymer (CFRP) laminates were used to externally strengthen the shear spans of the beam, under a commercial name of SikeWarp. Hex–230C. CFRP is available in rolled of 0.12 mm effective thickness, 300 mm width, and of about 5000 mm length. According to the data provided by the CFRP supplier, the fabrics had an elastic modulus of 231000 MPa, tensile (rupture) strength of 4100 MPa, and rupture strain of 1.7%.
– An epoxy mortar of about 2 mm thickness was applied to all strengthened beams as a substratum to the CFRP sheets, under a commercial name of
Sikadur–21. Compressive, bending and tensile strength as well as young’s modulus of such epoxy are 75, 25, 10 and 9000 MPa respectively.

Fig. 1: Details of test specimens.
2.3 Application of CFRP

Surfaces of the beam to be strengthened were roughened using a grinder, and the corners of the beam where the CFRP U–jackets were applied had been rounded in curved shape of about 30 mm diameter to reduce the stress concentration generated on the composite at the beam corners. After that, the concrete surfaces were cleaned by compressed air. An epoxy mortar (Sikadur–41) of about 2.0 mm thickness was applied to bonding surfaces as substratum to the CFRP sheets, but before that a primer coat (Sikadur–31) was applied first on the bonding surface to promote the adhesion between the concrete surface and the applied epoxy mortar. After about 24 hours a two–part epoxy adhesive (Sikadur–330) was applied in a thin layer over the epoxy mortar and the precut CFRP sheets were placed over it. The sheets were pressed firmly and rolled uniformly by a plast roller to squeeze out excess epoxy and all air bubbles.

2.4 Test Procedure

All beams were tested under four–points loading over a span of 1940 mm. The load was applied to the beams in increments. At each increment, the mid–span deflection, and the strains in the middle height of some of the CFRP U–strips were measured by means of dial and electrical strain gauges. The crack initiation and propagation were monitored by visual inspection during testing.

3. TEST RESULTS AND DISCUSSION

3.1 Failure Modes

In general, and as expected all test specimens failed mainly as a result of diagonal tension cracking (shear failure). Cracking pattern at ultimate load and failure modes of all beams are shown in Fig. (2). Each specimen exhibited an initial flexural crack in the region of pure bending and subsequent additional flexural cracks formed in the central region. As the applied load was increased a number of flexural shear and shear cracks were developed along the shear spans and one of them extended diagonally upward toward the loading point. Failure of control beam (B–1) was sudden and by diagonal tension. In case of strengthened beams, the diagonal tension failure was preceded by CFRP strips bond failure and/or CFRP rupture, and the diagonal crack occurred at a relatively higher load than for the control beam. All strengthened beams failed by concrete splitting and crushing behind the fiber strips. The splitting of concrete behind the strips caused these fiber strips to be ruptured or pushed out wards (debonding). In case of beam B–5 with 30 mm strip width, rupture of fiber strips along the path of the main diagonal crack was observed and there was no debonding of the fiber strips.
Fig. 2: Crack patterns of tested beams.
3.2 Load–Deflection Curves

Load–midspan deflection curves for all specimens are shown in Figs. (3 to 5). It can be noticed that, the initial slope of all curves remains identical. This means that the provided external shear reinforcement (U–strips) did not increase the initial flexural stiffness of the beam, but has a significant effect on both ultimate load and ductility. Figures (3 and 5) show that both fiber height and fiber spacing have a significant effect on load–carrying capacity and ductility of beams. Meanwhile for beams strengthened with the same fiber shear reinforcement ratio ($\rho_f$), increasing the number of strips and consequently decreasing the strip width have a slight effect on load–carrying capacity and ductility of beams (Fig. 4). Beam B–4 strengthened with nearly full U–warp, strips of fiber width equals b/2 and fiber spacing equals $W_f + 0.22d$ respectively showed the highest ultimate load and ductility among all test specimens.

![Fig.3: Influence of fiber effective height on load central deflection curve.](image)

3.3 CFRP Strains

Figures (6, 7 & 8) show the load versus vertical strain in carbon fiber sheet at mid–depth of the sheet at certain locations (See Fig. 1). Also the maximum strain ($\varepsilon_{\text{max}}$) recorded in these strips just before failure of beams are given in Table (3). From these figures and the results shown in the table, the following observations can be made:
Fig. 4: Influence of both fiber width and fiber spacing for the same ($\rho_l$) on load-central deflection curve.

Fig. 5: Influence of fiber spacing or fiber ratio ($\rho_l$) on load-central deflection curve.
**Fig. 6**: Load versus vertical carbon fiber strain.

(Effect of fiber depth)
Fig. 7: Load versus vertical carbon fiber strain (Effect of $W_f$ and $S_f$ for the same $\rho_f$)
Strain increment was very small in the pre-diagonal crack range. Once the diagonal cracks were formed a rapid increase in strains were recorded.

Beam B–4 with nearly full U–warp showed a higher strain at failure in compression with beams B–2 and B–3 with less sheet depth.

The maximum CFRP strains (ε_{\text{max.}}) measured just before failure of beams with nearly full U-warp reached values from 4050 to 6500 microstrain. These values are equal to about 24 to 38% of the CFRP rupture strain (ε_{fu} = 1.7%). The maximum strain value recorded by other [7] was in the range of 4000 to 6000 microstrain. This means that the ratio of the effective strain (stress) in the CFRP strips to its ultimate tensile strain (stress) is approximately little more than the range of 0.24 to 0.38. Therefore an upper limit value of 0.5 can be suggested for this ratio. This is in agreement with that mentioned before [13].

The values of the measured strains in the fibers crossing the path of the formed diagonal crack are approximately the same especially near failure. This means that the load carried by the CFRP strips crossing the shear crack is approximately uniformly distributed among these fibers. This observation is in agreement with that mentioned early by other [7].

**Fig.8:** Load versus vertical carbon fiber strain. (Effect of $S_f$ or $\rho_i$)
– For beams strengthened with the same $\rho_f$, neither the number of strips nor strip width has a significant effect on the value of the maximum measured CFRP strain.

### 3.4 Failure Load (Shear Capacity)

In case of beams strengthened with CFRP–U–strips, diagonal crack was always followed by CFRP debonding and/or rupture, and failure occurred at a load significantly higher than that for un-strengthened beam. The increase in failure load was ranged from 15.2% to 92.9% over that of the control beam and was depending on the effects of the following parameters.

– **Effective bonded height (depth) of CFRP strips ($h_{fp}$):**

Shear capacity of test specimens increased with the increase in the effective height of strips. In addition to that the relation between percentage increase in ultimate shear capacity and $W_f \cdot h_{fe}/S_f$ is approximately linear (Fig. 9-a). Beam B–4 with nearly full U–warp showed the highest shear strength among all beams compared to the control beam (92.9%). This means that for a shear strengthening of RC beam with CFRP–U strips to be effective it should ensured that, strip height should extend up to the maximum possible section depth (full depth).

– **Number of strips ($N_f$) and strip width ($W_f$) for the same ($\rho_f$):**

First it has to be mentioned that the number of strips and the strip width are obviously related to each other for any given fiber reinforcement ratio ($\rho_f$). Results showed that, for the same ratio of CFRP shear reinforcement ($\rho_f$), neither the number of strips ($N_f$) nor the strip width ($W_f$) has a significant influence on the ultimate shear capacity of the test specimens. Beam B–4 strengthened with strips of 60mm width ($b/2$) and spacing 120mm approx. [$W_f+(d/4)$] showed the heights increase in shear capacity (92.9%), while beam B–5 with $W_f = 30$ mm showed the smallest increase (82.6%). Beams B–6 and B–7 strengthened with fiber strips of 90 and 120 mm width respectively failed at a load almost 85.8% higher than the control beam. It has to be mentioned that beam B-6 had a fiber spacing $S_f = 0.67d$, while beam B-7 had a fiber spacing $S_f = 0.89d$

– **Fiber spacing ($S_f$) or amount of CFRP ($\rho_f$):**

Figure (9-b) showed that shear capacity of test beams increased approximately linearly as the fiber width/fiber spacing increased for constant value of $h_{fe}$ ($h_{fe}=240\text{mm}$). For example beams strengthened with fiber of 60 mm width and of 120, 180, and 240 mm fiber spacing showed an increasing in their ultimate shear capacity of about 92.9, 52.2 and 36.9% over that of the control beam respectively. Deniaud *et al.* [7] reported an increase of 94% for T–beams.
strengthened in shear with CFRP U–strips of a width of 50 mm and of gap of 50 mm.

The above results showed that beam strengthened with nearly full U–warp strips of width b/2 and of fiber spacing of about [W_f + (d/4)] showed the heights shear strength among all beams compared to the control beam. It has to be mentioned that Khalifa et al. [13] reported that the gap between two strips should not exceed (d/4), i.e. S_f ≤ W_f + (d/4). In addition to that the UK Concrete Society [14] proposed a spacing limit of the lesser of 0.8 d and W_f + (d/4).

### 3.5 CFRP Contribution to Shear Capacity

The shear capacity of RC beams strengthened using externally bonded CRFP U–strips can be calculated using the following expression, which based on truss analogy according to ACI procedures:

\[ V_u = V_c + V_s + V_f \]

In which \( V_c \) is the shear strength of concrete, \( V_s \) is the shear strength of steel stirrups and bent bars, and \( V_f \) is the contribution of CFRP–U strips to shear capacity of beam. \( V_c \) and \( V_s \) can be calculated according to provisions in existing design codes. However the main differences between available models lie in the evaluation of FRP contribution (\( V_f \)).
The value of \( V_f \) many be estimated from the summation of forces in CFRP strips intersecting the critical shear crack at ultimate limit state. Hence the CFRP contribution to shear capacity can be written as follows:

\[
V_f = 2 f_{fe} t_f W_f \frac{h_{fe} (\cot \beta + \cot \alpha_f) \sin \alpha_f}{S_f}
\]

Where \( t_f, W_f, h_{fe}, \) and \( S_f \) are the thickness, width, effective height, and center-to-center spacing of CFRP U-strips in mm. \( \beta \) and \( \alpha_f \) is the crack angle and fiber orientation with respect to longitudinal axis of beam (in this study \( \beta = 45^\circ \) and \( \alpha_f = 90^\circ \)). \( f_{fe} \) (or \( E_f \cdot \varepsilon_{fe} \)) is the effective stress of bonded CFRP U-strips at failure and this is the only unknown in the above equation to be determined for completing the analysis on CFRP contribution to shear capacity.

### 3.6 Effective Stress in CFRP Fabric \( (f_{fe}) \)

As mentioned above the prediction of the shear contribution of external CFRP reinforcement basically depends on the determination of the effective FRP stress \( (f_{fe}) \) or strain \( (\varepsilon_{fe}) \). The modeling of this effective stress or strain depends on several aspects, such as shape of shear crack, concrete strength, FRP strengthening method and bond length, and stiffness of FRP strips. These aspects and their interactions are very difficult to model and need extensive research which are currently not available. In addition to that the equations given by both draft of the Egyptian Code [17] and Chen et al.[4] shear strength models failed in predicting the contribution of CFRP–U strips to shear capacity of the tested beams (See Table 3). Therefore, the statical regression analysis of the experimental data of the beam tests with the aid of the models reported by others [4, 15 and 16] have been used to produce a simple expression for estimating the value of \( (f_{fe}) \) at the complete debonding of the critical CFRP–U strips as follows:

\[
f_{fe} = R \ f_{fu} , \text{ MPa}
\]

\[
R = \frac{0.4 K^2 (f_{cu})^{2/3}}{(E_f \ t_f)^{0.58} \varepsilon_{fu}} \leq 0.5
\]

where

\[
K = \frac{h_{fe} - L_e}{h_{fe}}
\]

\[
L_e = \sqrt{\frac{E_f \ t_f}{(0.8 f_{cu})^{0.5}}} , \text{ mm}
\]
Where: R is the ratio of the effective stress in the CFRP-U strips at failure of beam to the ultimate (rupture) tensile strength of the used fiber. An upper limit of 0.5 was suggested for this ratio before [4], and the results of this research supported this value. $\varepsilon_{fu}$ is the rupture strain of FRP, $L_e$ is the effective bond length, and K is the bonded length ratio. It has to be mentioned that the value of $h_{fe}$ should not exceed a value of 0.9 d. This is because the shear crack ends at a distance of 0.1 d below the compression face of the beam [13]. $E_f$ and $f_{cu}$ are in MPa.

After computing the effective stress (or strain) the contribution of CFRP–U strips to the shear capacity of beams can be calculated as follows:

$$V_f = 2 f_{fe} t_f \frac{W_f h_{fe}}{S_f}, \text{ kN}$$

### 3.7 Comparison with Experiments

Table (3) shows a comparison between the computed ($V_{f\text{pre.}}$) and experimental ($V_{f\text{exp.}}$) values of shear contribution of CFRP–U strips to the shear capacity of beams collected from this work and from the limited existing literature [1,2 and 4]. It has to be mentioned that the experimental $V_{f\text{ exp.}}$ is obtained by subtracting the shear capacity of the control beam from the capacity of strengthened beams, owing to the little difference of concrete strength among all tested beams. It can be seen that the proposed model can estimate the shear contribution of CFRP–U strips for all strengthened beams with satisfactory accuracy. It has to be mentioned that the above model has been derived from a limited experimental database. Thus it is essential that more experiments should be carried out taking into consideration more parameters as number of FRP layers, concrete strength, type and orientation of fiber, shear span to depth ratio and size of beam, and amount of internal shear reinforcement.

### CONCLUSIONS

Based on the results of this study, behavior of concrete beams without internal shear reinforcement strengthened in shear with CFRP–U strips, the following conclusions can be drawn:

1. CFRP–U strips appear to be a viable means of increasing the shear capacity of RC beams considerably. This increase depends strongly on height and amount or spacing of fiber strips according to the studied parameters.

2. A maximum increase in shear strength of beam of about 93% was obtained for the beam strengthened with nearly full U–warp strips of width equals to half the beam width ($W_f = b/2$) and of spacing center–to–center equals [$W_f + (d/4)$].
3. For beams strengthened with the same fiber shear reinforcement ratio ($\rho_f$), increasing the number of fiber ($N_f$) and consequently decreasing the strip width ($W_f$) has a slight effect on both ultimate load carrying capacity and ductility of the strengthened beams.

4. The contribution of CFRP–U strips to the shear capacity of beams increases almost linearly with the increase in the value of $W_f \cdot h_{ef} / S_f$ for the studied parameters.

5. For beams with nearly full U–warp the maximum strain CFRP measured just before failure is approximately in the range of 0.24 to 0.38 of the ultimate tensile strain of the fiber, i.e. the ratio of effective stress in the CFRP-U strips to its ultimate strength is approximately little more than the range of 0.24 to 0.38. An upper limit of 0.5 for this ratio was suggested, and the present results confirmed this limit.

6. A simple model is proposed to predict the contribution of CFRP–U strips to the shear capacity of strengthened beam, without internal shear reinforcements which fail in shear by CFRP debonding.

### Research Recommendations:

- For a shear strengthening of RC beam with CFRP–U strips to be effective it should ensure that, strip height should extend up to the maximum possible section depth (full depth), strip width not less than half beam width, and strip spacing (enter–to–enter) does not exceed the lesser of $[W_f + (d/4)]$, 0.7d and $W_f + 200$ mm.
Since the proposed model has been derived from a limited experimental data. Thus it is essential that more experiments should be carried out taking into consideration more parameters such as: number of FRP layers, concrete strength, type and orientation of fiber, shear span to depth ratio and size of beam, and amount of internal shear reinforcement.

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بالبحث مقارنة مقاومة القص القصوى التي تساهم بها الألياف والتي تم الحصول عليها عملياً لهذه الكرات بتلك المحاسبة طبقاً لمعادلات الكود المصري وتلك المعطاة بواسطة Chen التي تم استخلاصها ما يلي:

- تدعيم الكرات في منطقة القص بهذه التقنية يؤدي إلى زيادة مقومتها القصوى بنسبة حوالي 93%.

- ويحسن من سلوكها.

- لنفس نسبة تسليح القص فإن زيادة عدد شرائح التقنية وبالتالي تقليل المسافة بينها له تأثير ضئيل على مقاومة القص القصوى ومطيولية هذه الكرات.

- نسبة الانفعال (الإجهاد) الحادث بشرائح التقوية والمقاس قبل الانهيار مباشرة إلى الانفعال الأقصى (إجهاد الشد الأقصى) لتلك الشرائح يتراوح بين 24% إلى 38%.

- تم استنباط معادلات للتتبؤ بالقيم التي تساهم بها شرائح التقوية في مقاومة القص القصوى لهذه الكرات، وقد أثبتت هذه المعادلات كفاءة عند تطبيقها على الكرات الحالية وتلك المختبرة بواسطة آخرين.

- تم في نهاية البحث إعطاء بعض التوصيات الهامة للقائمين على أعمال التدعيم وكذا للدراسات المستقبلية.