ELASTIC-PLASTIC INVESTIGATION ON EFFECTIVENESS OF TWO DIMENSIONAL FUNCTIONALLY GRADED MATERIALS IN REDUCTION OF THERMAL STRESSES

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(Received April 7, 2008 Accepted April 17, 2008)

The two-dimensional functionally graded materials, (2D-FGMs) have been introduced in order to reduce the thermal stresses in machine elements that are subjected to severe thermal loading. No work was found that investigated the elastic-plastic stress analysis for 2D-FGMs. In the current work, a 3D finite element model of 2D-FGM plates made of ZrO₂, 6061-T6 and Ti-6Al-4V with temperature dependent material properties has been proposed to perform such analysis. An elastic plastic stress-strain relation based on the rule of mixture of the 2D-FGM has been introduced. Also, a 3D finite element model of conventional FGM plates, of ZrO₂/Ti-6Al-4V and ZrO₂/6061-T6, with temperature dependent material properties has been proposed for the investigation of these plates too. Then, elastic-plastic stress analysis of the considered four plates (two conventional FGMs and two 2D-FGMs) under the same thermal loading was carried out. It was found that heat conductivity of the metallic constituents of FGM has great effect on the temperature distributions. Minimum temperatures variation and minimum stresses can be obtained using ZrO₂/6061-T6/Ti-6Al-4V 2D-FGM. Also, the results indicate that only ZrO₂/6061-T6/Ti-6Al-4V 2D-FGM can stand with the adopted severe thermal loading without cracking or fracture.

KEYWORDS: 2D-FGM, rules of mixture, volume fractions, elastic-plastic material model, temperature dependent material properties, thermal stresses, thermal and structure finite element analysis.

1. INTRODUCTION

Recently, functionally graded material (FGM) has been introduced as a thermal barrier materials. In the last decade, such materials have been used in many industrial applications that are subjected to large difference of operating temperatures. Day by day FGMs prove their high capability as high temperature resistant materials and quietly gain this position, as described by Noda [1]. These materials overcome the drawbacks of the multi-layers composite plates [1,2] that have been commonly used as thermal barriers in machines and equipments such as cracking and separations through the interface.
The temperature distributions in machine elements that are used in several applications such as space shuttles, nuclear reactors, aircrafts, etc, change in two or three directions. Thus, proper operation of such elements necessitates the use of effective high-temperature resistant materials. Steinberg [3] showed the variations of the temperature at various places on the outer surface of a new aerospace craft during sustained flight at a speed of Mach 8 and altitude of 29 km. The temperature on the outer surface of such a plane ranges from 1033 K along the top of the fuselage to 2066 K degrees at the nose. Furthermore, this temperature level has to decay severely, through the thickness of the craft body, to the room temperature inside the craft. This added a new challenge to introduce and develop more effective high-temperature resistant materials that can withstand high-external temperatures that have variations in two or three directions. To overcome such problem Callister [4] suggested several different thermal protection materials to be considered in the design that satisfy the required criteria for specific regions of the spacecraft surface. However, Callister proposed design has the same drawbacks of the composite layers. Such drawbacks were overcome by using FGMs [1,2]. In 2003 Colombia space shuttle was lost in a catastrophic break up [5]. The speculation of such failure is that some kind of structural damage caused by outer surface insulation that fell loose when the Columbia lifted off [5]. Hindustan Times [6] reported that, damage to the space shuttle’s protective thermal tiles is emerging as a key focus of the probe into the Colombia tragedy. It is worth mentioning that conventional FGM may also not be so effective in the design of such advanced machine elements that are subjected to high temperature variations in two or three directions. Therefore, if the FGM has two-dimensional dependent properties, more effective high-temperature resistant material can be obtained.

Recently, many investigations [7-13] for 2D-FGM have been carried out. Unfortunately, all of them have considered exponential functions for continuous gradation of the material properties. The use of exponential functions for the material properties usually facilitates the analytical solution but don’t give real representation for material properties, except at the upper and lower surfaces of FGM. Aboudi et al. [14, 15] studied thermo-elastic/plastic theory for the response of FGM in two directions. Their studies circumvented the problematic use of the standard micro-mechanical approach that employed in the analysis of FGMs. The response of symmetrically laminated plates subjected to temperature change in one dimension was investigated by Aboudi et al. [14, 15]. They found that it is possible to reduce the magnitude of thermal stress concentrations by a proper management of the microstructure of FGMs. Cho et al. [16] have optimized the volume fractions distributions of FGM for relaxing the effective thermal stresses. They obtained the optimal volume fractions distribution in two directions for the FGM. The obtained optimum volume fractions have a random distribution, which is very difficult to represent or simulate as that of conventional FGM. Goupee and Vel [17] proposed a methodology for the two-dimensional simulation and optimization of material composition distribution of FGM. The two-dimensional quasi-static heat conduction and thermo-elastic problems were analyzed using the element-free Galerkin method. They obtained the spatial distribution of ceramic volume fraction by piecewise bi-
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The obtained optimum volume fractions have also a random distribution. From the investigations of Cho et al. [16] and Goupee and Vel [17] it can be noticed that their investigations aimed at optimizing the volume fractions distributions of conventional FGMs. Their results indicated that the obtained optimal volume fractions are randomly distributed in two directions.

The concept of adding a third material constituent to the conventional FGM to withstand the severe thermal stresses was introduced by Nemat-Alla [18]. The obtained material has been considered as 2D-FGM. For which the rules of mixture and the volume fractions relations have been also introduced. Comparison between the 2D-FGM and conventional FGM was carried out and showed that 2D-FGM has high capability to reduce thermal stresses than conventional FGM. It is worth mentioning that the temperature independent properties and elastic behavior of the FGMs have been considered through most of the above mentioned investigations. Therefore, realistic investigations of 2D-FGMs should be carried out considering the temperature dependent properties and elastic-plastic behavior of the 2D-FGMs.

Recently, a new method for manufacturing of FGMs via inkjet color printing has been reported by Wang and Shaw [19]. According to this method Al₂O₃ and ZrO₂ aqueous suspensions were stabilized electrostatically and placed in different color reservoirs in inkjet cartridges. The volume and composition of the suspensions printed in droplets at a small area were controlled by the inkjet cyan–magenta–yellow– black color printing principle. The proposed method shows the potential for the manufacturing of FGMs with arbitrarily 2D and 3D composition profiles. Therefore it can be properly applied to manufacture the 2D-FGM model proposed by Nemat-Alla [18].

The main aim of the current work is to investigate the 2D-FGM under two-dimensional severe thermal loading with consideration of the temperature dependence of their properties and elastic-plastic behavior. To achieve this objective a 3D finite element model of 2D-FGM plates made of ZrO₂, 6061-T6 and Ti-6Al-4V with temperature dependent nonlinear material properties is proposed and applied. Also, an elastic- plastic stress-strain relation based on the rule of mixture of the 2D FGM is included in the investigation. For the sake of comparison a 3D finite element model of conventional FGM plates made of ZrO₂/Ti-6Al-4V and ZrO₂/6061-T6 with temperature dependent material properties has been considered and applied in the present work. Then, an elastic-plastic stress analysis of the considered plates under the same transient thermal loading cycle that consists of heating followed by cooling is carried out. The obtained results in both cases are presented and compared in order to investigate and assess the effectiveness of 2D-FGM under severe thermal loading conditions.

2. FUNCTIONALLY GARDED MATERIAL MODELING

2.1 Volume Fractions of 2D-FGM

Consider a 2D-FGM plate of width \( w \) and thickness \( h \) as shown in Fig. 1. Through the current investigation \( x \), \( y \) and \( z \) coordinates that coincide with the directions of the cubic interpolation of volume fractions defined at a finite number of grid points. The obtained optimum volume fractions have also a random distribution. From the investigations of Cho et al. [16] and Goupee and Vel [17] it can be noticed that their investigations aimed at optimizing the volume fractions distributions of conventional FGMs. Their results indicated that the obtained optimal volume fractions are randomly distributed in two directions.
width, thickness and length of the FGM plates respectively. According to Nemat-Alla [18] the 2D-FGMs are made of continuous gradation of three distinct materials at least one of them is ceramic and the others are metallic alloys. The volume fractions of the 2D-FGM plate at an arbitrary location (point A) are expressed as [18]:

\[
\begin{align*}
V_1 &= \left[1 - \left(\frac{y}{h}\right)^{m_x} \right] \left[1 - \left(\frac{x}{w}\right)^{m_y}\right] \\
V_2 &= \left[1 - \left(\frac{y}{h}\right)^{m_x} \right] \left(\frac{x}{w}\right)^{m_y} \\
V_3 &= \left(\frac{y}{h}\right)^{m_x} \left(\frac{x}{w}\right)^{m_y}
\end{align*}
\]

(1) (2) (3)

Fig. 1. Geometrical parameters and coordinate system of the 2D-FGM plate.

where \(V_1\), \(V_2\) and \(V_3\) are the volume fractions of the three basic constituents of the 2D-FGM. The terms \(m_x\) and \(m_y\) are nonhomogenous parameters that represent the composition variations in \(x\) and \(y\) directions respectively. The compositions of the 2D-FGM adopted in the present analysis changes through the thickness \(h\) from 100% \(\text{ZrO}_2\), material 3, at the upper surface, \(y = h\), to a FGM of two different metals on the lower surface, \(y = 0\). The FGM of two different metals at the lower surface of the plate also changes from 100% 6061-T6 aluminum alloy, material 1, at left corner, \(x = 0\) and \(y = 0\), to 100% Ti-6Al-4V, material 2, at the right corner, \(x = w\) and \(y = 0\). This case of 2D-FGM is designated as \(\text{ZrO}_2/6061-\text{T6/Ti-6Al-4V}\). Also, another version of 2D-FGM may be obtained by interchanging the locations of material 1, 6061-T6, and material 2, Ti-6Al-4V. This case of 2D-FGM is designated as \(\text{ZrO}_2/\text{Ti-6Al-4V}/6061-\text{T6}\).

### 2.2 Rules of Mixtures of the 2D-FGM

The material properties at different positions within the 2D-FGM plate with porosity have been calculated using the rules of mixture that introduced by Nemat-Alla [18] where the thermo-mechanical properties are expressed as follows;
\[ \nu = v_1 V_1 + v_2 V_2 + v_3 V_3 \quad (4) \quad E = \frac{E_{oy} (1 - p_y)}{1 + \frac{p_y (5 + 8v)(37 - 8v)}{8(1 + v)(23 + 8v)}} \quad (5) \]
\[ \rho = \rho_o (1 - p_t) + \rho_a p_t \quad (6) \quad \lambda = \frac{\lambda_{oy} \left\{ (1 - p_y^{2/3}) \lambda_y + p_y^{2/3} \lambda_a \right\}}{p_y^{1/3} \lambda_a + (1 - p_y^{1/3}) \left\{ (1 - p_y^{2/3}) \lambda_{oy} + p_y^{2/3} \lambda_a \right\}} \quad (7) \]
\[ C = C_o (1 - p_t) + C_a p_t \quad (8) \]
\[ \alpha = \alpha_o (V_1 + V_2) + \alpha_a V_3 + \frac{V_3 (V_1 + V_2) (\alpha_u - \alpha_v) (K_o - K_y)}{K_y (V_1 + V_2) + K_y V_3 + (3K_y K_o / 4G_o)} \quad (9) \]

The porosity \( p_t \) is defined as;
\[ p_t = A_y \left( \frac{y}{h} \right)^{m_1} (p_y - p_x) + p_x \quad (11) \]

where \( A_y, p_x, \) and \( p_y \) are known functions [18]. Also, \( \lambda_a \) and \( C_o \) are the heat conductivity and heat capacity of air. \( v, E, \lambda, \alpha, \rho, C \) and \( \sigma_y \) are Poisson’s ratio, modulus of elasticity, heat conductivity, and coefficient of thermal expansion, density, heat capacity and yield stress. \( E_{oy}, \lambda_{oy}, \alpha_o, K_o, G_o, \rho_o \) and \( C_o \) are constants functions as given in details in [18].

### 2.3 Temperature Dependent Material Properties

The thermo-mechanical properties of ZrO₂, 6061-T6 aluminum alloy and Ti-6Al-4V which are the constituents of the proposed 2D-FGM are taken from the data given in references [20-23]. Those properties for each constituent are expressed as follows:

**[ZrO₂]**

\[ \lambda = 1.71 + 0.21 \times 10^{-3} T + 0.116 \times 10^{-6} T^2 \quad \text{[W/ (m \cdot °K)]} \]
\[ c = 2.74 \times 10^2 + 7.95 \times 10^{-1} T - 6.19 \times 10^{-4} T^2 + 1.71 \times 10^{-7} T^3 \quad \text{[J/(kg \cdot °K)]} \]
\[ \rho = 3.6570 / (1.0 + \alpha(T - 300.0))^3 \quad \text{[kg/m³]} \]
\[ \alpha = 13.31 \times 10^{-6} - 18.9 \times 10^{-9} T + 12.7 \times 10^{-12} T^2 \quad \text{[1/°K]} \]
\[ E = 132.2 - 50.3 \times 10^{-3} T - 8.1 \times 10^{-6} T^2 \quad \text{[GPa]} \]
\[ \nu = 0.333 \]
\[ \sigma_{ut} = 148.1 + 1.184 \times 10^{-3} T - 31.4 \times 10^{-6} T^2 \quad \text{[MPa]} \]
\[ \sigma_{uc} = 3136.09 / 146.0 \sigma_{ut} \quad \text{[MPa]} \]
\[ \sigma_{yt} = \sigma_{ut} \]
\[ \sigma_{yc} = \sigma_{uc} \]

**[Ti-6A1-4V]**

\[ \lambda = 1.1 + 0.017 T \quad \text{[W/(m \cdot °K)]} \]
\[ c = 3.5 \times 10^2 + 8.78 \times 10^1 T - 9.74 \times 10^4 T^2 + 4.43 \times 10^7 T^3 \quad \text{[J/(kg \cdot °K)]} \]
\[ \rho = 4420.0 / (1.0 + \alpha(T - 300.0))^3 \quad \text{[kg/m³]} \]
\[ \alpha = 7.43 \times 10^{-6} + 5.56 \times 10^{-9} T - 2.69 \times 10^{-12} T^2 \quad 300 K \leq T \leq 1100 K \quad \text{[1/°K]} \]
\[ \alpha = 10.291 \times 10^{-6} \quad 1100 K \leq T \quad \text{[1/°K]} \]
\[ E = 122.7-0.0565T \]  
\[ \nu = 0.289 + 32.0 \times 10^6T \]  
\[ \sigma_{yt} = 2.4339972 \times 10^6T^3 + 1.606979 \times 10^4T^2 - 1.110047725366T + 1540.9254 \]  
\[ \sigma_{yc} = \sigma_{yt} \]  
\[ \sigma_{ut} = 1252.01 \times 0.8486T \]  
\[ \sigma_{uc} = \sigma_{ut} \]  
\[ n = -1.479 \times 10^{-12}T^3 - 4.189532 \times 10^{-6}T^2 + 6.570936 \times 10^{-4}T + 7.45382661 \times 10^{-2} \]  
\[ k = -1.1529137818T + 2306.143627023 \]  
\[ \lambda = 220 \]  
\[ c = 0.5203T + 643.9 \]  
\[ 300K \leq T \leq 980K \]  
\[ c = 1160 \]  
\[ 800K \leq T \]  
\[ \rho = 2715.0/[1.0 + \alpha (T-300.0)]^3 \]  
\[ \alpha = 0.02 \times 10^{-6}T + 1.72 \times 10^{-3} \]  
\[ 300K \leq T < 950K \]  
\[ \alpha = 33.21 \times 10^{-6} \]  
\[ 950K < T \]  
\[ E = 9.7222 \times 10^{-8}T^3 - 1.809524 \times 10^{-4}T^2 + 6.17659 \times 10^{-2}T + 67.107143 \]  
\[ \nu = 5E-05T + 0.335 \]  
\[ 300K \leq T < 950K \]  
\[ \nu = 0.5 \]  
\[ 950K \geq T \]  
\[ \sigma_{yt} = -2.2351 \times 10^{-11}T^6 - 8.78356 \times 10^{-7}T^3 + 2.392895 \times 10^{-3}T^2 - 2.0939133T + 686.65931 \]  
\[ \sigma_{yc} = \sigma_{yt} \]  
\[ n = 1.05E-12T^4 - 3.52167 \times 10^{-9}T^3 + 4.3713 \times 10^{-6}T^2 - 2.382033 \times 10^{-3}T + 0.493957143 \]  
\[ k = -8.442 \times 10^{-12}T^5 + 3.49766 \times 10^{-8}T^4 - 5.71743 \times 10^{-5}T^3 + 4.631645368 \times 10^{-2}T^2 - 18.72634T + 3153.88998 \]  

where \( \sigma_{yt}, \sigma_{yc}, \sigma_{ut}, \sigma_{uc}, k \) and \( n \) are tensile yield strength, compressive yield strength, ultimate tensile stress, ultimate compressive stress, strength coefficient and strain hardening exponent respectively.

It is noteworthy that the thermo-mechanical properties that designated by * in the front of them are represented graphically in references [21,23] and curve fitting approximations were carried out to express them in the formulae given above.

### 2.4 Elastic-Plastic Material Model for 2D-FGM

Recently, the stress-strain power law hardening model was adopted for representing the elastic-plastic behavior of the conventional FGMs in the investigations reported in [24-26]. In those investigations the material properties were evaluated using the volume fractions and rules of mixtures. In the current investigation the stress-strain power law hardening model will be derived and used in the elastic-plastic analysis for 2D-FGM. The effective stress, \( \sigma \), and effective strain, \( \varepsilon \), of the 2D-FGM that is composed of three constituents material are expressed as functions of the basic constituents effective stresses, effective strains and volumes fractions as follows:
\[ \sigma = \sum_{i=1}^{3} \sigma_i V_i \quad (12) \]
\[ \varepsilon = \sum_{i=1}^{3} \varepsilon_i V_i \quad (13) \]

The stress-strain power law hardening model can be written as:
\[ \sigma = k(\varepsilon_o + \varepsilon)^n \quad (14) \]
where \( k, n \) and \( \varepsilon_o \) are the strength coefficient, strain hardening exponent and initial strain of the composite materials, respectively. The initial strain \( \varepsilon_o \) can be expressed as:
\[ \varepsilon_o = \left( \frac{\sigma_y}{k} \right)^{\frac{1}{n}} - \frac{\sigma_y}{E} \quad (15) \]
where \( \sigma_y \) and \( E \) are the initial yield stress and modulus of elasticity of the composite materials.

The strength coefficient, \( k \), and strain hardening exponent, \( n \), of the composite materials are expressed as functions of the basic constituents strength coefficients, strain hardening exponent and volume fractions as follows:
\[ k = \sum_{i=1}^{3} k_i V_i \quad (16) \]
\[ n = \sum_{i=1}^{3} n_i V_i \quad (17) \]

Using equations (10) and (12) to (17) the effective stress-strain relation of the composite materials can be expressed as:
\[ \sigma = \left( \sum_{i=1}^{3} k_i V_i \right) \left[ \left( \sum_{i=1}^{3} \sigma_y i V_i \right)^{\frac{1}{\sum_{i=1}^{3} n_i V_i}} - \sum_{i=1}^{3} \sigma_y i V_i \left[ 1 + \frac{p_y (5 + 8\nu)(37 - 8\nu)}{8(1 + \nu)(23 + 8\nu)} \right] \right] \frac{\sum_{i=1}^{3} n_i V_i}{E_{oy}(1 - p_y)} + \sum_{i=1}^{3} \varepsilon_i V_i \quad (18) \]

where \( E_{oy} \) and \( p_y \) are previously defined functions as mentioned above. Also, \( \nu \) is Poisson’s ratio of the composite materials as defined by equation (4).

Fig. 2. The applied non-uniform heat flux on the upper surface of the 2D-FGM plate.
3. TRANSIENT THERMAL LOADING

The thermal loading considered in the present study was achieved by subjecting the 2D-FGM plate which was initially at a uniform temperature of 300 °K to a nonuniform heating heat flux on the upper surface \((y = h)\) as shown in Fig. 2. The applied nonuniform heat flux has the following form:

\[
q = (q_{\text{max}} - q_{\text{min}}) \sin \left( \frac{\pi}{2} \left( 1 - \frac{x}{w} \right)^m \right) + q_{\text{min}}
\]  

(19)

where \(q_{\text{max}}\) and \(q_{\text{min}}\) are the maximum and minimum values of the heat flux and \(m\) is a constant. Values of \(q_{\text{max}}\) 400 kW/m\(^2\) and \(q_{\text{min}}\) 200 kW/m\(^2\) are found to be quite enough to check the effectiveness of the 2D-FGM plate in relaxing thermal stresses through the current investigation, since Kokini and Case [27] used a constant heat flux of 335 kW/m\(^2\) to initiate surface edge crack in functionally graded thermal barrier coating.

The lower surface of the plate, \((y = 0)\), is subjected to cooling by convection to an ambient temperature of 300 °K. The right and left surfaces of the plate, \((x = 0, x = w)\), are thermally isolated. According to Choules and Kokini [2] and Kokini and Case [27] a convection heat transfer coefficient \(h_L = 1000\) W/m\(^2\)°K and 300 °K ambient temperature is adopted. After the heating stage reaches a steady state condition the upper surface is left to cool down by convection to the ambient temperature of 300 °K. The cooling heat transfer coefficient at the upper surface, \(h_{Uc}\) is taken to be 1000 W/m\(^2\)°K according to Kokini and Case [27]. It is worth mentioning that cooling stage starts at time \(\tau_{ss}\) after reaching steady state of heating stage, i.e during cooling \(\tau > \tau_{ss}\). This time has been estimated in the model by checking the steady state condition by estimating the temperature difference at each time step.

The transient temperature, \(T(x,y,\tau)\) variation over the plate in \(x\) and \(y\) directions is determined by solving the following transient two dimensional heat equation using finite element method:

\[
\frac{\partial \lambda}{\partial x} \frac{\partial T(x,y,\tau)}{\partial x} + \frac{\partial \lambda}{\partial y} \frac{\partial T(x,y,\tau)}{\partial y} + \lambda \left( \frac{\partial^2 T(x,y,\tau)}{\partial x^2} + \frac{\partial^2 T(x,y,\tau)}{\partial y^2} \right) = \rho c \frac{\partial T(x,y,\tau)}{\partial \tau}
\]

(20)

The corresponding thermal boundary conditions for heating and cooling stages are as follows:

\[
T(x,y,0) = 300K \quad \text{initial condition}
\]

(21)

\[-\lambda \frac{\partial T(x,y,\tau)}{\partial y} \bigg|_{y=h} = q \quad \text{for heating stage at the top surface}
\]

(22)

\[
\lambda \left. \frac{\partial T(x,y,\tau)}{\partial y} \right|_{y=0} = h_L \left[ T(x,0,\tau) - 300 \right] \quad \text{for heating and cooling stages at the lower surface}
\]

(23)

\[
\left. \lambda \frac{\partial T(x,y,\tau)}{\partial y} \right|_{y=h} = h_{Uc} \left[ T(x,0,\tau) - 300 \right] \quad \text{for cooling stage at the top surface}
\]

(24)

The isolated thermal boundary conditions at the right and left plate surfaces are;

\[
\lambda \left. \frac{\partial T(x,y,\tau)}{\partial x} \right|_{x=0} = 0
\]

(25)

\[
\lambda \left. \frac{\partial T(x,y,\tau)}{\partial x} \right|_{x=w} = 0
\]

(26)
4. FINITE ELEMENT MODEL
The assignment of material properties in the computational model should reflect the property variation within the FGM structure that will be simulated. It is worth mentioning that significant efforts have been done in previous researches to implement the continuous variation of the FGM properties in the finite element formulations. Le et al. [28] indicated that implementation of the continual spatial variation of the properties of nonhomogenous materials in the finite element formulation does not present a computational problem since the stiffness matrix may be determined by averaging across each element. Generally there are two methods that can be adopted to account for the material properties in the finite element formulation. Either through assignment of the properties for each element individually or dividing the whole structure into numerous areas then assigning properties to each area [29-31]. Santare and Lambros [32] introduced a formulation for calculation of material properties in graded elements which automatically interpolates the material properties within the element. Also, Li et al. [28] and Kim and Paulino [33] have proposed a generalized isoparametric formulation in their application of the finite element for materials with an internal property gradient. In these studies mean properties, calculated by integration within each element, were used for the stiffness matrix. Rousseau and Tippur [34] have introduced a finite element technique involves definition of properties as function of temperature then assignment of temperature values to nodes. This finite element formulation leads to an interpolation of temperatures within the elements, this result in a continuous variation in properties.

In the current investigations temperature dependent thermo-mechanical properties are taken into consideration through the finite element analysis. In order to obtain more accurate predictions of thermal stresses that are expected to be induced in the adopted FGM plates the thermo-elastic-plastic behavior of the constituent materials should be considered. When the upper ceramic surface of FGM plate is subjected to thermal cyclic load (heating followed by cooling) thermal stresses will develop inside the plate. This is attributed to the difference in the coefficient of thermal expansion from point to point inside the plate. The thermo-mechanical properties are calculated based on volume fractions and the rules of mixture of FGM as described in [18].

The investigated FGM plates have a thickness $h$ of 30 mm, a width $w$ of 300 mm and a length of 700 mm. The 3D finite element model, used in the current computations of the coupled elastic-plastic thermo-mechanical problem, contains 40320 eight-nodded thermal-solid brick elements. This number of elements results from uniform dividing of the FGM into 240 elements through plate width, 24 elements through plate thickness and 7 elements through plate length. The mesh is refined in the plate cross section, x-y plane, (240x 24 elements) because of the material nonhomogeneity in this plane such refinement is expected to guarantee more accuracy of the results. On the other hand, a small number of elements have been taken along the plate length, because of the material homogeneity along that direction. The mechanical boundary conditions used in the finite element model are as follows; all nodes at the plate bottom surface are roller supported to prevent their movement in the
y-direction. Also nodes at the plate left side surface (x = 0) and the plate mid section (z = 0) are prevented from movement in the x and z-direction respectively.

The numerical solutions of the present investigation have been carried out for each of the four adopted cases of FGM plates, two 2D-FGM and two conventional FGM, according to the following steps: 1- The plate is numerically subjected to a non-uniform heating from the surrounding medium followed by sudden cooling. 2- The transient temperature distribution is obtained at discrete time increments by the solution of the thermal problem equations (20-26). 3- The resulting displacements and thermal stresses were determined by the solution of the 2D-FGM thermal stresses problem under the predetermined temperature distribution as described above. 4- The stresses on each node were averaged according to the stresses on its associated elements and shape function. It is noteworthy that the values of the nonhomogenous parameter \( m_y \) are same for all adopted four cases. Also, the nonhomogenous parameter \( m_x \) is zero for the conventional FGMs since there is no variation of the composition in the x-direction.

Assuming that the plate deformations did not affect the temperatures, the problem was modeled as a quasi-static thermo-mechanical problem.

5. RESULTS AND DISCUSSIONS

The 2D-FGM plates, \( \text{ZrO}_2/6061\text{-T6/Ti-6Al-4V} \) and \( \text{ZrO}_2/\text{Ti-6Al-4V/6061-T6} \), and conventional FGM plates, \( \text{ZrO}_2/\text{Ti-6Al-4V} \) and \( \text{ZrO}_2/\text{6061-T6} \), are investigated under the same thermal loads. The obtained results for different cases are analyzed and compared as follows.

5-1 Temperature Distributions

The temperature distributions obtained for the adopted four plates and the maximum temperatures that occur after reaching the steady state of the heating stage of the thermal loading are presented in Fig. 3. From Fig. 3-a it can be noticed that for \( \text{ZrO}_2/\text{Ti-6Al-4V} \) conventional FGM plate the temperature variation on its upper ceramic surface, \( \text{ZrO}_2 \), ranges from 2070 °K to 1480 °K. Also, there is a large temperature difference between the upper and lower surfaces of the plate, where the maximum and minimum temperatures are found to be 2070 °K and 500 °K respectively. For \( \text{ZrO}_2/\text{6061-T6} \) conventional FGM plate, Fig. 3-b indicates that the temperature variation on its upper ceramic surface, \( \text{ZrO}_2 \), ranges from 1180 °K to 790 °K. As for the temperature differences between the upper and lower surfaces of these plates, where the maximum and minimum temperatures exist, Fig. 3-b shows that these temperatures are about 1180 °K and 500 °K respectively. This low temperature levels for the second case of conventional FGM plate may be attributed to the fact that the 6061-T6 aluminum alloy has higher heat conductivity compared with Ti-6Al-4V alloy.
Fig. 3. Maximum variations of temperature distributions that were achieved after reaching steady state during heating stage.

From Fig. 3-c it can be noticed that, for ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate, the temperature variation on upper ceramic surface, ZrO$_2$, ranges from 1120 °K to 1360 °K. It is noticeable that the high temperature, 1360 °K, is found at the upper right corner where the heat flux is minimum, $q_{\text{min}}$ (Fig.2), while the lower temperature, 1120 °K, is found at the upper left corner where the heat flux is maximum, $q_{\text{max}}$ as shown in Fig. 2. This may be attributed to the fact that the 6061-T6 Aluminum alloy at the left side of the plate has higher heat conductivity compared with Ti-6Al-4V alloy on the right side surface. The maximum and minimum temperatures are of about 1360 °K and 480 °K respectively as shown in Fig.3-c. Finally, from Fig. 3-d it can be noticed that for ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate the temperature vitiation on the upper ceramic surface, ZrO$_2$, ranges from 2090 K to 790 K. This temperature difference is the largest difference on the upper surface of the four adopted cases considered in the current investigation. These results indicate that this plate is the worst one among the adopted four plates because of the large differences of the temperature on it. This may be attributed to the fact that the 6061-T6 aluminum alloy
has higher heat conductivity compared with Ti-6Al-4V alloy and their unsuitable positioning in 2D-FGM relative to the location of \( q_{\text{max}} \) and \( q_{\text{min}} \). Also, the difference between the maximum temperature, 2090 K, on the upper surface and the minimum temperature, 503 K, on lower surface is the largest compared to the corresponding difference on the other plates.

![Figure 4](image)

Fig. 4. Maximum temperature distribution on the upper surface that achieved after heating stage reaches steady state versus the normalized length, \( x/w \).

Figure 4 shows a comparison between the temperature variations on the upper ceramic surface, \( \text{ZrO}_2 \), for the adopted four cases after reaching steady state. It is clear that the temperature variations have distribution corresponding to the variation of the heat flux that applied on the upper surface except for the case of \( \text{ZrO}_2/6061\)-T6/Ti-6Al-4V 2D-FGM plate. The corresponding here means that the maximum and minimum temperatures on the upper surface occur very close or at locations of the maximum and minimum heat fluxes, respectively as can be seen from Figs. 2 and 4. As for the \( \text{ZrO}_2/6061\)-T6/Ti-6Al-4V 2D-FGM plate Fig. 4 indicates that the minimum temperature occurs at the location of maximum heat flux while maximum temperature occurs on the left side i.e. at minimum heat flux. This may be attributed to the fact that the 6061-T6 aluminum alloy, set at the right side where the heat flux is maximum, has high heat conductivity and consequently low thermal resistance that increases the amount of heat transferred in the places subjected to maximum heat flux. Moreover, the resulting temperature difference on the upper surface of this plate is the lowest difference compared with other plates. Thus, these results indicate that such arrangement of metal constituents of 2D-FGM represents the optimum as far as heat transfer is compared.

### 5-2 Thermal Stresses

The induced thermal stresses due to the applied thermal loads are calculated and normalized by their corresponding yield stresses. The yield stresses are calculated at different positions as a function of the corresponding temperatures. Through the current investigations it is considered that cracks initiations and propagations will occur when the normalized equivalent stresses are tensile and greater than unity. Since von Mises equivalent stress criterion could not distinguish between compressive and tensile stresses, the first principle stresses will be used to indicate that the equivalent stresses are compressive or tensile.
Elastic-Plastic Investigation on effectiveness of Two

5-2-1 Thermal Stresses during Heating Stage

From the current investigation it was found that the maximum values of the thermal stresses during heating stage were achieved when reaching steady state of heating process. Therefore, the maximum values of the normalized equivalent stresses \( \sigma_{eq}/\sigma_Y \), which were achieved after reaching steady state of heating process, were calculated and presented in Fig. 5 for the adopted four cases. Figure 5-a shows the variations of the maximum values of the normalized equivalent thermal stresses during heating process in \( \text{ZrO}_2/\text{Ti-6Al-4V} \) conventional FGM plate. It can be seen from this figure that for \( y/h \geq 0.85 \) the normalized equivalent stresses are greater than unity. This means that cracks are expected to occur in such places if the normalized equivalent stresses are tensile. The variations of the normalized first principle stresses in \( \text{ZrO}_2/\text{Ti-6Al-4V} \) FGM plate were calculated and it was found that the normalized first principle stresses in the region \( y/h \geq 0.85 \) are compressive and their values ranges from \(-0.259\) on the upper surface, \( y = h \), to about \(-0.02\) at \( y/h \approx 0.85 \). Thus, cracks will not occur in this plate during heating stage. Figure 5-b shows the distribution of the...
normalized equivalent stresses, which were achieved after reaching steady state of heating process, in ZrO$_2$/6061-T6 conventional FGM. It is noticeable that for $y/h \leq 0.25$ the maximum values of the normalized equivalent stresses are greater than unity. This means that cracks initiation and propagation may occur in such places. From the calculations of the normalized first principle stresses in that region, which were achieved after reaching steady state of heating process, it was found that they are compressive and their values range from -0.2 to -1.4. Since the compositions in the region $y/h \leq 0.25$ for ZrO$_2$/6061-T6 FGM plate are metallic rich, which means that it possesses ductile behavior in that region, and the maximum values of the normalized equivalent stresses are compressive. Thus, no cracks will initiate in this plate and just localized plastic deformations will be induced. Figure 5-c shows the variations of the maximum values of the normalized equivalent stresses, which achieved after reaching steady state of heating process, in ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate. It is noticeable that for $0.15 \leq y/h \leq 0.28$ and $x/w \leq 0.13$ the maximum values of the normalized equivalent stresses are greater than unity. This means that cracks initiation and propagation may occur. From the calculations of the normalized first principle stresses, which were achieved after reaching steady state of heating process in that region, it was found that they are compressive. Since the compositions in the region $0.15 \leq y/h \leq 0.28$ and $x/w \leq 0.13$ for ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate is metallic rich, which means that it has ductile behavior in that region, and the maximum values of the normalized equivalent stresses are compressive, thus, no cracks will initiate in this plate and just localized plastic deformations will be induced. Figure 5-d shows the variations of the normalized equivalent stresses, which were achieved after reaching steady state of heating process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate. It is noticeable that for $y/h \geq 0.8$ and $x/w \leq 0.5$ the maximum values of the normalized equivalent stresses are greater than unity. Also, the normalized first principle stresses, which were achieved after reaching steady state of heating process, in this region ($y/h \geq 0.8$ and $x/w \leq 0.5$) are compressive. This means that no fracture or crack initiations will occur.

Finally, from the above results and discussions a general conclusion can be obtained which is that compressive stresses were developed in the adopted four cases during heating stage. This means that the developed stresses during heating stage are not critical.

5-2-2. Thermal Stresses during Cooling Stage

5-2-2-1 Equivalent thermal stresses during cooling stage

From the current investigation it was found that the maximum values of the thermal stresses during cooling stage were achieved after starting the cooling process. Figure 6 shows the variations of the maximum values of the normalized equivalent stresses ($\sigma_{eq}/\sigma_Y$), which were achieved after starting of the cooling process, for the adopted four cases. Figure 6-a shows the variations of the maximum values of the normalized equivalent stresses in ZrO$_2$/Ti-6Al-4V conventional FGM plate. It is noticeable that in the region $y/h \geq 0.9$ and $x/w \leq 0.9$ the normalized equivalent stresses are greater than unity. This means that cracks initiations and propagations will occur in such region if the normalized equivalent stresses were tensile. From the current calculation it was
found that in the region $y/h \geq 0.9$ and $x/w \leq 0.9$ the normalized first principle stresses, which were achieved after starting of the cooling process, were tensile and their values ranges from 1.0 to 1.3. Therefore, it can be concluded that cracks will initiate and propagate nearly all over the upper ceramic surface.

![Graph](image)

**Fig. 6.** Variations of the maximum values of the normalized equivalent thermal stresses, which were achieved after starting of the cooling process.

Figure 6-b shows the variations of the maximum values of the normalized equivalent stresses, which were achieved after starting of the cooling process, in ZrO$_2$/6061-T6 conventional FGM plate. It is noticeable that for $y/h \leq 0.11$ and $x/w \leq 0.65$ the normalized equivalent stresses are greater than unity. This means that fracture or localized plastic deformation may occur in such plates. From the calculations of normalized first principle stresses, which were achieved after starting of the cooling process, it was found that for this region $y/h \leq 0.11$ and $x/w \leq 0.65$ these stresses are tensile and their values range from 1.0 to 1.4. This means that cracks will initiate at the lower left corner of the plate in the region $y/h \leq 0.11$ and $x/w \leq 0.65$. Figure 6-c shows the variations of the maximum values of the normalized equivalent stresses, which were achieved after starting of the cooling process, in ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate. It is noticeable that yielding will not occur, since the value of the
normalized equivalent stresses does not exceed unity. Figure 6-d shows the variations of the maximum values of the normalized equivalent stresses, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate. It is noticeable that for $y/h \geq 0.87$ and $x/w \leq 0.35$ the normalized equivalent stresses are greater than unity. From the calculated values of the normalized first principle stresses, which were achieved after starting of the cooling process, it found that these stresses are tensile and their values range from zero to 1.2. This means that cracks will initiate and propagate in that region.

From the above results and discussion it was found that cracks will be induced in the adopted cases except for the case of ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM. Also, these results indicate that the thermal stresses induced during the cooling stage are tensile. The results in case of ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM, Fig. 6-c, indicate that the maximum values of the normalized equivalent stresses through most of this plate are zero especially on the upper ceramic surface. This represents an advantage for this plate over other types that makes it preferable as thermal barrier material.

![Graph](image)

Fig. 7. Variations of the normalized stresses components, which were achieved after starting of the cooling process, in conventional ZrO$_2$/Ti-6Al-4V FGM.

5-2-2-2 Estimation of cracks direction during cooling stage
From the above discussion it was found that cracks will occur in the adopted cases except for the case of ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM. In order to estimate the crack initiation and propagation direction the distribution of the normal thermal stresses components should be investigated. The variations of the normalized stresses $\sigma_x/\sigma_Y$, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V conventional FGM plate are shown in Fig. 7-a. It is noticeable that for $y/h \geq 0.91$ and $x/w \leq 0.9$ the normalized stresses $\sigma_x/\sigma_Y$ vary from 1.2 to 1.3. Figure 7-b shows the variations of the normalized stresses, $\sigma_y/\sigma_Y$, which were achieved after starting of the cooling process, in conventional ZrO$_2$/Ti-6Al-4V FGM plate. It is noticeable that the variations of the normalized stresses $\sigma_x/\sigma_Y$ are very similar in distributions and magnitudes to $\sigma_y/\sigma_Y$, Fig. 7-a. The maximum and minimum values of the normalized stresses $\sigma_y/\sigma_Y$, which were achieved after starting of the cooling process, that resulted
in this plate were estimated to be 0.14 and -0.16 respectively. This means that there is no crack initiation in the x-direction. Generally, from Fig. 7 and the calculated values of the normalized equivalent stresses $\sigma_{eq}/\sigma_{Y}$, which were achieved after starting of the cooling process, it can be concluded that vertical surface cracks will be initiated in ZrO$_2$/Ti-6Al-4V FGM plate under the prescribed thermal loads on the upper surface. These cracks are expected to extend in the region $y/h \geq 0.91$ and $x/w \leq 0.9$ in the y-direction.

Figure 8-a shows the variations of the normalized stresses components, which archived after starting of the cooling process, in conventional ZrO$_2$/6061-T6 FGM plate. It is noticeable that for $y/h \leq 0.1$ and $x/w \leq 0.65$ the normalized stresses $\sigma_{x}/\sigma_{Y}$ are equals or greater than unity with variations similar to the variations of the normalized equivalent stresses $\sigma_{eq}/\sigma_{Y}$, Fig. 6-b. The normalized stresses $\sigma_{x}/\sigma_{Y}$ reaches 1.2 at the lower surface which means that surface crack in y-direction will initiate in that place. Figure 8-b shows the variations of the normalized stresses $\sigma_{y}/\sigma_{Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/6061-T6 FGM plate. It is noticeable that for $y/h \leq 0.1$ and $x/w \leq 0.65$ the normalized stresses $\sigma_{y}/\sigma_{Y}$ are greater than unity. Also, the normalized stresses $\sigma_{y}/\sigma_{Y}$ reaches 1.4 at the lower left corner which means that vertical surface crack in this place will initiate. Also, from the calculated values of the normalized stresses $\sigma_{y}/\sigma_{Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/6061-T6 conventional FGM plate it was found that maximum and minimum values are 0.15 and -0.75 respectively. This means that there is no crack initiation in the x-direction. Finally, from Figs. 8 and the calculated values of the normalized stress $\sigma_{y}/\sigma_{Y}$ it can be concluded that vertical surface cracks will initiated in ZrO$_2$/6061-T6 FGM plate in the lower surface and will be extended to the region $y/h \leq 0.1$ and $x/w \leq 0.4$ in the y-direction.

Figure 9-a shows the variations of the normalized stresses $\sigma_{x}/\sigma_{Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate. It is noticeable that yielding or crack initiations will not occurs since the value of the normalized equivalent stresses does not exceed unity, as shown in Fig. 6-c. Fig.
9-b, also, shows the variations of the normalized stresses $\sigma_z/\sigma_Y$ in ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate, which archived after starting of the cooling process. The calculated values of the maximum and minimum normalized stresses $\sigma_z/\sigma_Y$ are 0.03 and -0.065 respectively which are elastic values. From Figs. 6 to 9 and the calculated values of the normalized stresses $\sigma_y/\sigma_Y$ it is clear that ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plate can stand the applied thermal loading without crack initiation or fracture or localized plastic deformation.

Fig. 9. Variations of the normalized stresses components, which archived after starting of the cooling process, in ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM plat.

Fig. 10. Variations of the normalized stresses components, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate.
Figure 10-a shows the variations of the normalized stresses $\frac{\sigma_x}{\sigma_Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate. It is noticeable that for $y/h \geq 0.87$ and $x/w \leq 0.35$ the values of the normalized stresses $\frac{\sigma_x}{\sigma_Y}$ are greater than unity. This means that crack initiations and propagations will take place in the $y$-direction. Also, Fig. 10-b shows the variations of the normalized stresses $\frac{\sigma_y}{\sigma_Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate. It is noticeable that the variations of the normalized stresses $\frac{\sigma_y}{\sigma_Y}$ are very similar in distributions and magnitudes to the normalized stresses $\frac{\sigma_x}{\sigma_Y}$. Also, it was found that the calculated values of normalized stresses $\frac{\sigma_y}{\sigma_Y}$, which were achieved after starting of the cooling process, in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate have maximum and minimum values of 0.1 and -0.6 respectively which means that there is no crack initiation in the $x$-direction. Generally, from Fig. 10 and the calculated values of the normalized stresses $\frac{\sigma_y}{\sigma_Y}$ it can be concluded that vertical surface cracks will initiate in ZrO$_2$/Ti-6Al-4V/6061-T6 2D-FGM plate on the upper surface. These cracks are expected to extend through the region $y/h \geq 0.87$ and $x/w \leq 0.35$ in $y$-direction.

Finally, from the above results and discussions it can be concluded that cracks initiation and propagation will occur in the adopted four cases in the $y$-direction in different zones for each plate, except the case of ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM. Where no cracks are expected just only a localized plastic deformation will occur in the lower left surface. This localized plastic deformation will be interpreted to compressive residual stresses.

6. CONCLUSIONS

From the results of the current investigation of 2D-FGM of different compositions subjected to a prescribed severe thermal loading using an elastic-plastic finite element model, adopting elastic-plastic strain hardening behavior, the following conclusions can be drawn:

1- The suitable distribution of the basic constituents of the 2D-FGM can reduce the value of the maximum temperature that is developed in the plate.

2- Heat conductivity of the metallic constituents of FGM has great effect on the temperature distributions that result from the thermal loads.

3- Maximum values of equivalent thermal stresses that are developed in the adopted four cases during the heating stage are compressive while they are tensile during the cooling stage.

4- Cracks may initiate and propagate during the cooling stage.

5- ZrO$_2$/6061-T6/Ti-6Al-4V 2D-FGM can stand with severe thermal loads.

6- The 2D-FGMS proof their high capabilities in reducing thermal stresses than conventional FGMS if suitable distributions of the basic constituents are employed.

REFERENCES


تأثير التصرف المرن اللدن على كفاة المواد المدّرجة وظيفياً ثنائية الأبعاد في تخفيض الإجهاد الحراري

المواد المدّرجة وظيفياً ثنائية الأبعاد (2D-FGM) قُدمت كبذل تكنولوجي لتقليل الإجهادات الحرارية في أجزاء الماكينات التي تُعرض إلى تحميل حراري حاد. والأبحاث السابقة لم تتم فيها دراسة تأثير التصرف المرن اللدن لهذه المواد. في الدراسة الحالية تم تقديم نموذج من العناصر المحدودة ثلاثية الابعاد لألواح من المواد المدّرجة وظيفياً احادية وثنائية الأبعاد مؤلفة من أكسيد الزركونيوم وسبائك الالمنيوم 6061-T6 و6V وTI-6AL-4V. وقد تم استنباط معادلات لتمثيل السلوك المرن اللدن لهذه المواد. بعد ذلك تم إجراء تحليل إجهادات مرن لدن لألواح من المواد المدّرجة وظيفياً احادية وثنائية الأبعاد تحت نفس التحميل الحراري. وقد وُجد أن معامل التوصيل الحراري للمواد الأساسية المكونة للمواد المدّرجة وظيفياً ثنائية الأبعاد له تأثير عظيم على توزيعات درجات الحرارة. وقد تم استنتاج أن الواح المواد المدّرجة وظيفياً ثنائية الأبعاد ZRO6061-T6/TI-6AL-4V هي الوحيدة القدرة على تحمل الجهات الحرارية القاسية بدون حدوث شروخ أو كسر.