A NON LINEAR OPTIMAL PID CONTROL OF A HYDRAULIC CRANE

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In this work, an adaptation-based method for deriving tuning guidelines for proportional-integral-derivative (PID) controllers that take explicitly into account the presence of nonlinear behavior is proposed. The central idea behind the proposed method is to adjust the PID controller parameters to force the nonlinear system response to follow a second order critically damped model reference (MR). The design method starts by using classical tuning guidelines (typically derived on the basis of linear approximations) to obtain reasonable bounds on the tuning parameters. A modifier technique will produce a correction signal to upgrade the PID controller signal to eliminate the relative error between the nonlinear system and the MR.

The efficiency of the proposed tuning method is demonstrated through application to a hydraulic crane which is inherently nonlinear and contains components exhibiting strong friction, saturation, variable inertia mechanical loads, etc. The characteristics of these non-linear components are usually not known exactly as structure or parameters. It is found that the MR-PID control policy provided the most consistent performance in terms of rise time and settling time with set-point changes regardless of the nonlinearities characteristics.

KEYWORDS: Optimal control, Nonlinear PID Control, Model Reference, Hydraulic Crane.

1. INTRODUCTION

The majority (over 90%) of the regulatory loops in the process industries use conventional proportional-integral-derivative (PID) controllers. Owing to the abundance of PID controllers in practice and the varied nature of processes that the PID controllers regulate, extensive research studies have been dedicated to the analysis of closed-loop properties of PID controllers and to devising new and improved tuning guidelines for the PID controllers, focusing on closed-loop stability, performance and robustness (see, for example, the survey papers [1], [2]). Most of the tuning rules are based on obtaining linear models of the system, either through running step tests or by linearizing a nonlinear model around the operating steady-state, and then computing values of the controller parameters that incorporate stability, performance and robustness objectives in the closed-loop system.

While the use of linear models for the PID controller tuning makes the tuning process easy, the underlying dynamics of many processes are often highly complex,
due, for example, to the inherent nonlinearity of the hydraulic systems, or due to operating issues such as time delays and disturbances. Ignoring the inherent nonlinearity of the process when setting the values of the controller parameters may result in the controller's inability to stabilize the closed-loop system and may call for extensive re-tuning of the controller parameters.

At the application side, the electrohydraulic drives are widely used in industrial applications, such as in rolling and paper mills, as actuators in aircraft, and in many different automation and mechanization systems. The main reason for their broad industrial applications is the great power capacity that they can exert (as compared to their DC or AC counterparts), while preserving good dynamic response and system resolution [3].

Systems containing fluid power components offer interesting and challenging applications of modern and classical control techniques. The use of microcomputers and many feedback devices for hydraulic drives allows for implementation of different control algorithms that result in better steady-state and dynamic performances in fluid power control systems. There are a number of research results on the applications of adaptive control [4], robust control [5], and variable structure control [6] in electrohydraulic control systems.

In the Sliding Mode Control (SMC) method, the system trajectory is forced to reach the sliding surface and to slide along it, or to remain in its vicinity [6, 7]. Since in many situations the SMC is found robust to a great extent to plant parameter variations or uncertainties in the model of the system to be controlled, it has found broad applications. However, the chattering is a signify problem in the SMC implementations and solutions that either reduce or eliminate it had been investigated in [8, 9].

In a parallel way, even though fuzzy logic controllers often produce results superior to those of traditional controllers [10, 11], the control engineer has found difficulties in accessing the fuzzy logic controllers because of the following limitations: The design of the fuzzy logic controller is not straightforward due to heuristics involved with control rules and membership functions. There is no standard systematic method for tuning the fuzzy logic controller parameters.

Ayman A. Aly and Aly S. Abo El-Lail, [12] recognized that a hydraulically actuated system contains a host of nonlinear elements, thereby making a linear controller ineffective. Furthermore, the authors illustrated that linearization of the dynamic equations over a small operating range and the design of an appropriate controller for each condition had limitations with time variant parameters challenge.

The aim of this paper is to design a new PID controller which has the ability to solve the control problem of highly nonlinear systems such as the hydraulic crane, which is shown in Fig.1. To test the designed optimal PID parameters, there are two ways one is verifying it practically and the other one is simulating the system with its full nonlinearities which will be the safer and the economizer solution. The proposed method has not local compensation for each type of the system nonlinearities. It deals with final system response and tries to correct it according to defined reference model.
2. SYSTEM CONSTRUCTION

The servo system is composed of a hydraulic power supply, an electrohydraulic servovalve, a cylinder, mechanical linkages, and control. The piston position of the cylinder is controlled as follows: Once the voltage input corresponding to the desired position is transmitted to the servovalve controller, the controller signal current is generated. Then, the valve spool position is changed according to the input current applied to the torque motor of the servovalve. Depending on the spool position and the load conditions of the piston, the rate as well as the direction of the oil supplied to each cylinder chamber is determined.

If it is necessary to represent servovalve dynamics through a wider frequency range, a second-order transfer function must be used. The relation between the servovalve spool position $x_v(t)$ and the input current $i_v(t)$ can be written as

$$\frac{d^2 x_v(t)}{dt^2} + 2\zeta_\nu \omega_\nu \frac{dx_v(t)}{dt} + \omega_\nu^2 x_v(t) = \omega_\nu^2 k_v i_v(t)$$

(1)

where $k_v$ represents the gain of the servovalve, $\omega_\nu$ is the natural frequency of the servovalve, and $\zeta_\nu$ is the damping ratio of the servovalve.

The valve spool occludes the orifice with some overlap so that for a range of spool positions there is no fluid flow. This overlap prevents leakage losses that increase with wear and tear. Thus, the dead zone should be placed between the valve dynamics and actuator/load dynamics. For the sake of simplicity, this dead zone is equivalently moved to the position between the output of the controller and the input current of the valve. So, the dead zone nonlinearity may be characterized as shown in Fig. 2 and approximately described as:
\[
\begin{align*}
  i(t) - I_1 & \quad \text{if } i(t) > I_1 \\
  0 & \quad \text{if } |i(t)| \leq I_1 \\
  i(t) + I_1 & \quad \text{if } i(t) < -I_1 
\end{align*}
\]

where \(i_c(t)\) is the current from the controller and \(I_1\) the width of the dead zone.

The equations of the servovalve flow to and from the actuator (assuming symmetric valve port, zero lap design and zero return pressure) are as follows,

For positive \(x_i(t)\):

\[
q_f(t) = C_d W x_i(t) \text{sgn}(P_s - P_f(t)) \left[ \frac{2}{\rho} P_s - P_f(t) \right],
\]

\[
q_n(t) = C_d W x_i(t) \text{sgn}(P_n(t)) \left[ \frac{2}{\rho} P_n(t) \right]
\]

For negative \(x_i(t)\):

\[
q_f(t) = C_d W x_i(t) \text{sgn}(P_f(t)) \left[ \frac{2}{\rho} P_f(t) \right],
\]

\[
q_n(t) = C_d W x_i(t) \text{sgn}(P_s - P_n(t)) \left[ \frac{2}{\rho} P_s - P_n(t) \right]
\]

where \(x_i(t)\) is the spool displacement, \(P_s\) is the supply pressure, \(\rho\) is the mass density of the oil, \(C_d\) is the discharge coefficient of the orifice, \(W\) is the width of the orifice, suffix \(n\) denotes the annular side and suffix \(f\) denotes the full side.

The linearized flow equation of the actuator is given by [12]:

\[
q_{le}(t) = K_l \frac{A_e}{A_f} \left[ 1 + \left( \frac{A_n}{A_f} \right)^2 \right] p_{le}(t) + A_e \ddot{X}_p(t) + \frac{2A_e}{A_f} \frac{P_{le}(t)}{4B} \left[ V_f(t) + \left( \frac{A_n}{A_f} \right)^2 V_n(t) \right]
\]

where

\[
p_{le}(t) = \frac{p_f(t)A_f - p_n(t)A_n}{A_e}, \quad q_{le}(t) = \frac{q_f(t) + q_n(t)}{2}, \quad A_e = \frac{A_f + A_n}{2},
\]

\(p_{le}(t)\) is the effective load pressure, \(q_{le}(t)\) is the effective load flow rate, \(A_e\) is the effective piston area, \(B\) is the oil bulk modulus, \(K_l\) is the leakage coefficient of the piston, \(X_p(t)\) is the piston displacement, \(V_{le}(t)\) is the oil volume under compression in the annular side of the cylinder, \(V_f(t)\) is the oil volume under compression in the full side of the cylinder, \(A_i\) is the annular area of the cylinder, \(A_f\) is the full area of the cylinder.

The equation of motion of the crane is given by

\[
P_{le}(t) A_e = M_e \dddot{X}_p(t) + B_e \dddot{X}_p(t) + F_d(t)
\]

where \(M_e\) represents the equivalent mass of both the variable inertia load and the piston, \(B_e\) is the equivalent viscous damping coefficient, and \(F_d(t)\) represents the disturbing forces like friction forces.

The various friction characteristics depend on lubrication, relative velocities of bodies at the contact point, pressures and others [3, 4]. A typical friction characteristic is presented in Fig.3.
The exact simulation of the nonlinear behavior of friction in the vicinity of a zero velocity is difficult. The friction force is approximately simulated by the stick-slip friction law. The value of the stick-slip friction for positive values of \( X_p(t) \) is given simply by

\[
F_f(t) = \begin{cases} 
F_{sl}(t) & \text{for } 0 \leq \dot{X}_p(t) < \Delta \dot{X}_p(t) \\
F_{st}(t) & \text{for } \dot{X}_p(t) \geq \Delta \dot{X}_p(t) 
\end{cases}
\] (7)

where \( F_{sl}(t) \) is the slip friction which is proportional to the piston speed \( \dot{X}_p(t) \) and \( F_{st}(t) \) is the stick friction. Inside this small region, surrounding \( \dot{X}_p(t) = 0 \), a necessary approximation is that consider \( F_f(t) \) to be zero.

The dynamics of hoses and pipes connecting the servovalve and the actuator are simulated by a time delay function. The transport lag function is given by

\[
H(s) = e^{-sT_d}
\] (8)

Transport delays are approximated by a first-order lag

\[
e^{-sT_d} \approx \frac{1}{\left( \frac{T_d}{2} \right)s + 1}
\] (9)

where \( T_d \) is the delay time. The approximation introduces an extra pole to the system transfer function, but, unlike Pade's method, it does not introduce an extra zero [4].

The actuator is equipped with one hydraulic accumulator in the supply port to cope with the dynamic flow demands. A linear variable differential transformer (LVDT) measures piston displacement with gain of 10 V/m. The block diagram of the hydraulic crane system is shown in Fig. 4. The system geometric transformation and its physical parameters are illustrated in Appendixes A and B.

### 3. CONTROLLER DESIGN

PID-controller is the most common in many industrial applications and it has been stated in many papers that a PID-controller has been used in hydraulic position servo systems [4, 5, 13, 14 and 15]. The most serious nonlinearities are the nonlinearities of valves, load, and friction forces. The nonlinearities in the forward loop as shown in Fig. 5 cause the position error in position servo systems, and they are responsible for
performance limitations. Unquestionably, the plant as a whole poses a very difficult dynamics to control.

As detailed model of the crane would be difficult to derive so it is too complex to be used in regulator design. The common solution is often at first approximate the real complex model to linearized one and finally adjust nearly the PID controller parameters according to standard design method such as Ziegler-Nichols [16] method which is considered in the most popular one during the last 50 years. The linear PID-control algorithm in Laplace form is presented as follow,

$$U(s) = \left[ K_P + \frac{K_I}{s} + K_Ds \right] E(s)$$  \hspace{1cm} (10)

where $U(s)$, $E(s)$ are the controller output, system response error, $K_P$, $K_I$, and $K_D$ are the proportional, integral and derivative PID gains respectively. According to the studies of many researchers in the field of fluid power control systems [5, 6, 7 and 10] the following conclusions can be made:

Linear PID-controllers are not suitable controllers for hydraulic position servo because of overshoots and limit cycles. Since PID controller parameters are usually designed using either one or two measurement points of the system frequency response as Ziegler-Nichols method, their control performance may not satisfy the desired time-response requirements.

Also linear PID controller with additional nonlinearities compensators face the fact that the system nonlinearities have variable structure and parameters which clear the needing to readjust the controller gains time to time.

When a system has different operating points with widely differing dynamic properties and high position accuracy is required, it is not always possible to control it with a fixed parameter controller, a nonlinear design of PID-controller might be a solution. A model reference control scheme can guide continuously the tuning of the controller parameters to deal with system regardless the nature of its nonlinearities will be our propose policy during this work.

Fig. 4 Block diagram of the hydraulic crane system.

Fig. 5. Model of the hydraulic drive system
The proposed scheme is shown in Fig. 6. The steps of the proposed control strategy are as follow.

**The First Stage:** Because of there is no standard design method for the PID control parameters with nonlinear system, it will be designed for the linearized one as a starting point. In order to have a good closedloop time response, the following performance function needs to be considered during the design of the PID controller parameters:

\[ J(K_P, K_I, K_D) = ITASE \]  

(11)

where ITASE is the integral time absolute square error of the system output. Thus, the optimal PID controller design problem may be stated as finding the PID controller parameters which give minimum performance function. This step output will be the initial PID parameters which will be implemented in the nonlinear system.

**The Second Stage:** is defining the reference model, in the time domain; specifications for a control system design involve certain requirements associated with the time response of the system. The requirements are often expressed in terms of the standard quantities on the rise time, settling time, overshoot, and steady-state error of a step response. The time response of a standard second-order system is widely used to represent the above time-domain requirements as a model reference to the real nonlinear time variant system. Thus, the second-order system is chosen for the tracking mode, whose transfer function is

\[ G_m(s) = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2} \]  

(12)

where the parameters \( \omega_m \) is the model natural frequency and \( \zeta_m \) the model damping ratio, which are chosen according to the desired time-domain response requirements of the closed-loop system.

**The Third Stage:** If the controller gains which were achieved in the first stage are implemented in the nonlinear model, the response will be worthier than the response of the linearized one. As shown in Fig.6 the system response is compared with the model reference response. The relative error in between of them will be fed to the modifier.
controller which will produce a correction signal to modify the optimal PID controller to eliminate the relative error.

Using modifying law similar to Equation (10), one obtains the rules base for the modifier controller. The variations in the PID gains will be as follows:

\[
\Delta K_p = \alpha(t) \cdot K_p, \quad \Delta K_I = \beta(t) \cdot K_I \quad \text{and} \quad \Delta K_D = \gamma(t) \cdot K_D
\]  

(13)

where \( \alpha(t) = K_{pm} \cdot e_r(t) \), \( \beta(t) = K_{lm} \int e_r(t) \, dt \), \( \gamma(t) = K_{dm} \frac{de_r(t)}{dt} \),

\( e_r(t) \) is the relative error between the MR and the system response, \( ce_r(t) \) is change in the relative error, \( \Delta K_p, \Delta K_D, \) and \( \Delta K_I \) are the PID gains variation, \( K_p, K_D, \) and \( K_I \) are the calculated gains of the PID controller based on Equation (11), \( \alpha(t), \beta(t) \) and \( \gamma(t) \) are the modifier outputs and \( K_{pm}, K_{lm} \) and \( K_{dm} \) are scaling factors.

4. RESULTS

By considering the fast system dynamics, the sampling period in the simulation routine is chosen to be 0.001 sec. The cost function is given by \( J(K_p, K_I, K_D) \). The reference input is a step signal, which changed from 0 to 10 degree. Using the MATLAB optimization toolbox, the optimal PID parameters \( K_p=34.286, K_I=0.686 \) and \( K_D=0.171 \) are found.

Figure 7.a shows the closed-loop responses due to step input of linearized and nonlinear model with the same optimal PID controller parameters which is designed based on the linearized model. The expected system response will be worthier than the response of the linearized one due to the effect of the nonlinearities which appears as increasing in the overshoots, rise time and settling time however, the steady state error is zero in the two cases. The corresponding controller signals for each case are shown in Fig.7.b.

Since in many industrial applications, it is necessary to assure that the response has minimum/no overshoot, this is achieved in Fig.8.a which, illustrates the model reference responses and the nonlinear system responses based on the proposed control policy with model damping ratio (\( \zeta_m = 1 \)) and two suggested model natural frequencies (\( \omega_m = 50 \) and \( 100 \) rad/sec). It is easy to decide the value of \( \zeta_m \) to be critically damped however the value of \( \omega_m \) need extra effort to be chosen within the physical limitation of the cylinder maximum velocity. It is noticed that the responses are improved compared with the responses of Fig.7.a where, there is smaller settling and rise times with no steady state error or overshoot. In the nonlinear PID controller design, the model reference controller outputs are used to adapt the final output of the controller according to the relative error in the performance of the reference model and the nonlinear system responses.

The parameters of the modifier controller are \( K_{pm}=1.45, K_{lm}=0.720 \) and \( K_{dm}=0.140 \). The crossholdings controller outputs of the proposed strategy for the tested model are illustrated in Fig.8.b. It is interesting to notice that the amplitude of controller’s signals became smaller with implementing the proposed strategy compared with Fig. 7.b, which is signifying index in the hydraulic system design and in its power energy saving.
Figure 8.c represents the modification signals in the PID control of the nonlinear system with the two tested models. It is a remarkable notice that the modification mechanism work only with the transient change in the system response. Another good application with a continuous motion test as an input signal for the nonlinear system with the proposed controller policy is shown in Fig.9. The system response follows the model reference with delay of 0.018 sec.
5. CONCLUSIONS

Nonlinear dynamic phenomena in hydraulic systems are unique and diverse. It is difficult to estimate their global nature from local nature by linear analysis. Thus, the hydraulic systems are often very conservatively tuned and the fact that the cost of getting the tuning wrong can be highly destructive and costly. To effectively assess the
performance of the proposed tuning method, the control system performance is evaluated via simulations.

In this paper the position control problem of a hydraulic crane is addressed. The highly nonlinear behavior of the system limits the performance of classical linear controllers used for this purpose. It has been demonstrated that the MR-PID control can be successfully implemented in the control system of a hydraulic crane. Since there are nonlinearities in the hydraulic position control system, it is difficult to achieve high-precision tracking performance using only linear PID controllers. The results obtained show that the proposed controller policy exhibits much better response, much better tracking characteristics, and retains excellent following motion property comparable to, or better, than that obtained by the conventional optimally tuned PID controller.

In addition to faster and more accurate responses, the proposed controller design steps are simple, thus, the application of the algorithm can be made wider than that of the conventional PID controllers.

6. REFERENCES

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Appendix-A
The Hydraulic Crane Geometric Transformation

For the kinematics analysis [3], the schematic representation of the hydraulic crane is illustrated in Fig.A.1.

Since the two vectors, \( \overrightarrow{r_{DB}} \) and \( \overrightarrow{r_{CD}} \) are of fixed length and are rigidly connected at point D, they are combined. The reference angle, \( \theta \), is adjusted to reflect the orientation of the combined vector and this adjusted value is termed, \( \theta^* \). This is shown graphically in Fig.A.2.

![Fig. A.1. Linkage dimensions of consequence](image1)

Application of the coordinate system indicated in Fig.A.2 yields,

\[
l_{AB} \left( \hat{i} \cos \alpha + \hat{j} \sin \alpha \right) + l_{BC} \left(-\hat{i} \cos \theta^* - \hat{j} \sin \theta^* \right) + l_{CA} \left( - \hat{j} \right) = 0
\]

(A-1)

The length of the actuator, \( l_{AB} \), as a function of the angular displacement of the boom, \( \theta^* \), is then found through substitution of Equation A.2 into Equation A.1:
Also a similar velocity analysis is performed in order that the angular velocity of the crane arm could be related to the velocity of the hydraulic actuator as follows:

\[ i_{AB} = \dot{\theta} l_{BC} \sin \left[ \frac{\pi}{2} - \sin^{-1} \left( \frac{l_{BC}}{i_{AB} \sin \left( \frac{\pi}{2} + \theta^* \right)} \right) \right] - \theta^* \]  

(A-4)

Appendix-B
System Physical Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque motor gain of servovalve</td>
<td>( k_v )</td>
<td>3.75x10^{-4}</td>
<td>m/mA</td>
</tr>
<tr>
<td>Natural frequency of servovalve</td>
<td>( \omega_v )</td>
<td>1068</td>
<td>rad/sec</td>
</tr>
<tr>
<td>Damping ratio of servovalve</td>
<td>( \zeta_v )</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Rated flow rate of servovalve</td>
<td>( Q_v )</td>
<td>0.333x10^{-3}</td>
<td>m^3/sec</td>
</tr>
<tr>
<td>Total leakage coefficient</td>
<td>( C_t )</td>
<td>1.0x10^{-10}</td>
<td>m^5/(sec N)</td>
</tr>
<tr>
<td>Supply pressure</td>
<td>( P_s )</td>
<td>14x10^6</td>
<td>Pa</td>
</tr>
<tr>
<td>Bulk module of oil</td>
<td>( B )</td>
<td>7.0x10^8</td>
<td>Pa</td>
</tr>
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<td>Mass density of oil</td>
<td>( \rho )</td>
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</tr>
<tr>
<td>Equivalent mass of both the load and the piston</td>
<td>( M_e )</td>
<td>100.0</td>
<td>kg</td>
</tr>
<tr>
<td>Diameter of rod</td>
<td>( d_{rod} )</td>
<td>0.12</td>
<td>m</td>
</tr>
<tr>
<td>Diameter of piston</td>
<td>( d_{piston} )</td>
<td>0.14</td>
<td>m</td>
</tr>
<tr>
<td>Maximum stroke of cylinder</td>
<td>( X_p )</td>
<td>1.2</td>
<td>m</td>
</tr>
<tr>
<td>Length of pipeline and hoses from pump to cylinder</td>
<td>( L )</td>
<td>5.0</td>
<td>m</td>
</tr>
<tr>
<td>Cylinder Coulomb friction force</td>
<td>( F_{fc} )</td>
<td>200</td>
<td>N</td>
</tr>
</tbody>
</table>

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\[ i_{AB} = \frac{l_{BC} \cos \theta^*}{\cos \left( \alpha \tan \left( \frac{l_{BC} \sin \theta^* + l_{AC}}{l_{BC} \cos \theta^*} \right) \right)} \]

(A-3)

PID التحكم

أمثل لاخطٍى في رافعة هيدروليكية

في هذا العمل، تم اقتراح طريقة أساسها إشتقاق تعليمات ضبط للحاكم التفاضلي النسبي التكاملية (PID) التي تأخذ في إعتبارها بشكل واضح وجود السلوك اللاخطي. إن الفكرة المركزية وراء الطريقة المقترحة هو أن تُعدّ معاللات الحاكم PID لإجبار إستجابة النظام اللاخطي لمتاربة إشارة مرجعية لنموذج من الدرجة الثانية (MR) ذو خمدين حر يُعتبر إشتقاق مدينة بإستعمال طريقة الضبط الكلاسيكية (إشبُّط نموذجياً على أساس التقريرات الخطية) للحصول على حدود معقولة في ضبط المعاللات. تقنية التدعيم ستنتج إشارة تصحيح لرَّقية إشارة الحاكم PID لإزالة الخطأ النسبي بين النظام اللاخطي والنموذج.
إن كفاءة طريقة الضبط المقترحة ظهرت خلال التطبيق إلى رافعة هيدروليكية و التي هي لأخطية أصلاً و تحتوي على مكونات تتغوص لإحتمال قوي و إشباع و أحال قصور ذاتي متغير الميكانيكية، الخ. إن خصائص هذه المكونات اللاخطية عادة ليست معروفة بالضبط كتركيب أو معاملات.

وقد وجد أن سياسة الحاكم ذات النموذج المرجعي قدمت الأداء الأكثر ثباتاً من ناحية وقت الارتفاع و وقت الاستقرار مع تغييرات نقطة الضبط بغض النظر عن الخصائص اللاخطية.