A NEW VOLTAGE COLLAPSE PROXIMITY INDICATOR

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ABSTRACT—Voltage stability problems have been one of the major concerns for electric utilities as a result of increase demanding of electric power. This paper develops a new voltage collapse proximity indicator using catastrophe theory together with comparative singularity of power flow Jacobian and modal analysis. The application of the proposed indicator has been demonstrated on multimachine power systems.

KEYWORDS: Voltage stability criteria, Catastrophe theory, Jacobian matrix, Modal analysis.

1 - INTRODUCTION

Recently a number of utilities in different countries throughout the world have experienced voltage collapse problems. In some cases blackouts occurred as a result of voltage collapse [1-4]. Part of the reason for these types of problems is that today's power system utilities are highly stressed and heavily loaded without an adequate corresponding increase in the system capacity.

The problem of collapse may be simply explained by an inability of the power system to supply the reactive power or by an excessive absorption of reactive power by the system itself. Reactive power problems arise in power systems under a variety of conditions. For lightly loaded system, too much reactive power may be injected into the network by shunt elements resulting in overly high voltages at the voltage uncontrolled buses. Alternatively, under heavy load conditions, there may be insufficient injected reactive power causing the voltage to drop. In some cases heavily loaded power systems, particularly when the system configuration comprises long transmission lines, the voltage drop caused by the dropping of a generator or a transmission line cannot be recovered even if the static capacitors at load ends are switched on. This type of abnormal voltage rapid fall is called voltage instability or voltage collapse phenomena. There are two general types of tools for voltage stability analysis: dynamic and static. Dynamic analysis uses time-domain simulation to solve nonlinear system differential/algebraic equations. Static analysis is based on the solution of conventional or modified power flow equations.

Dynamic analysis provides the most accurate replication of the time responses of power system [5]. Accurate determination of the time sequence of the different events loading to system voltage instability is essential for post-disturbance analysis.
and the coordination of protection and control. However, time-domain simulations are
time consuming in terms of CPU and engineering required for analysis of results. Also,
dynamic analysis does not readily provide information regarding the sensitivity of
degree of instability. These limitations generally make dynamic analysis impractical for
examination of a wide range for system conditions for determining stability limits.

Static analysis [6] involves only the solution of power flow equations and
therefore is computationally much more efficient than dynamic analysis. Several
algorithms have been developed to study the steady-state (static) voltage instability.
The minimum singular value of the system Jacobian matrix has been proposed as a
voltage collapse index [7]. However calculating the minimum singular value is time
consuming due to the high dimension of the Jacobian matrix. To improve the feasibility
of this method, a fast algorithm to compute the minimum singular value was proposed
[8]. Modal analysis also reported for voltage instability assessment [9]. This method
calculates a set of the smallest eigenvalues of the reduced Jacobian matrix and the
associated participation factors. The eigenvalues are used as voltage instability
indicators and the participation factors for weak area identifications. Voltage – Power
(real or reactive) sensitivity is another index for voltage collapse detection. During
normal operating conditions, the Voltage – Power sensitivity is a finite value, and it
will increase with the system loading. When voltage collapse occurs, the Voltage – Power
sensitivity will be infinite [10]. Another developed method using the distance in
the load parameter space between a given operating condition and the critical point,
which is the voltage collapse point, as index. Algorithms for calculating this distance
were further investigated, while this index can provide a load power margin for an
operating condition, which is particularly valuable to system operators, the
computational burden is a main concern [11].

The study of this phenomena could be put into framework of catastrophe
theory which describes how sudden jumps in the system states can arise from a smooth
change of the system parameters [12]. The voltage stability manifold is defined as an
equilibrium surface of critical points. It can be obtained by deriving the minimum of
the system function which involves voltage and system parameters as state and control
variables respectively. In reference [13], the theory was applied on a simple system
consisting of a single generator supplying a static load through a lossless transmission
line. The control parameter (a < 0) was used as an indicator for voltage stability index.

In this paper, an attempt has been made to find a physical explanation for
voltage instability phenomena in interconnected multi-machine system using the
catastrophe theory together with comparative singularity of power flow Jacobian and
modal analysis. The study is based on the analysis of the phenomena under the effects
of changing the system parameters.

2 - CATASTROPHE THEORY

In any system (physical or natural) if some of the parameters of that system
continue to vary very smoothly, a critical stage may be reached at which point the
system exhibits a sudden jump from state to another. The other state may be different
from the original state. Power system like many of the most interesting phenomena in
nature involves discontinuities. One of the reasons for these discontinuities is the
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voltage collapse. Catastrophe theory tries to deal with the properties of these discontinuities.

Consider a gradient system whose behavior is usually smooth, but sometimes (or in some places) exhibits discontinuation and described by a potential function $E_p$ with $n$ state variables ($X_1, \ldots, X_n$) and $m$ control variables ($C_1, \ldots, C_m$). Then the equilibrium equations are given by:

$$\frac{\partial E_p}{\partial X_i} = 0 \quad i = 1, \ldots, n$$

(1)

Define an $m$-dimensional equilibrium surface in the ($n + m$) dimensional space spanned by the $X_i$ and $C_k$. Since the potential depends upon the control parameters, the Hessian of $E_p$ and its eigenvalues also depend on these control parameters. For certain values of control parameters, one or more of these eigenvalues may assume the value zero. When this happens, the Hessian of $E_p$ is

$$\det \left[ \frac{\partial^2 E_p}{\partial X_i \partial X_j} \right] = \det [E_{ij}] = 0$$

(2)

and the system critical points are called non-isolated degenerate or non-Morse critical points at which the system exhibits instability, but when the $\det [E_{ij}] \neq 0$, the critical points are called isolated non-degenerate or Morse critical points at which the system is stable. Therefore the system potential function $E_p$ can be split into parts (stable/unstable part). This is called splitting lemma [14-16], because it allows us to split the critical point into two parts: part involved in structural stability (non-degenerate associated with the non-vanishing eigenvalues), and the other involved in instability (degenerate associated with the vanishing eigenvalues).

According to Thom's theorem, in an open neighborhood of a critical point (where $\ell$ eigenvalues vanish), the original potential function $E_p$ can be written in the canonical form.

$$E_p = \text{Cat}(\ell, m) + \sum_{j=\ell+1}^{n} \lambda_j y_j^2$$

(3)

Where, $\text{Cat}(\ell, m)$ is the catastrophe function

$\lambda_j$ is the number of non-zero eigenvalues

$y_j$ is the coordinates associated with non-vanishing eigenvalues

The catastrophe function $\text{Cat}(\ell,m)$ gives the qualitative configurations of the system discontinuities that occur in the neighborhood of critical points. The second term in equation 3 does not contribute for discontinuities and can be ignored in the analysis.

According to Thom's classification theory, in any system governed by a potential function, and in which the system behavior is determined by no more than four different control variables, only seven qualitative different types of catastrophe functions are possible and given in Table 1. These are called elementary catastrophes.
Table 1: Seven elementary Catastrophes.

<table>
<thead>
<tr>
<th>Catastrophe Manifold</th>
<th>Control variable</th>
<th>State variable</th>
<th>Function</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fold</td>
<td>1</td>
<td>1</td>
<td>$x^{3/3} + a, x$</td>
<td>$x^2 + a$</td>
</tr>
<tr>
<td>Cusp</td>
<td>2</td>
<td>1</td>
<td>$x^{3/4} + b, x^{2}/2 + ax$</td>
<td>$x^{1} + b, x + a$</td>
</tr>
<tr>
<td>Swallowtail</td>
<td>3</td>
<td>1</td>
<td>$x^{5}/5 + a, x^{3}/3 + bx^{2}/2 + cx$</td>
<td>$x^{3} + ax^{2} + bx + c$</td>
</tr>
<tr>
<td>Butterfly</td>
<td>4</td>
<td>1</td>
<td>$x^{6}/6 + a, x^{4}/4 + b, x^{3}/3 + cx^{2}/2 + dx$</td>
<td>$x^{7}/5 + a, x^{3}/3 + bx^{2}/2 + cx$</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>3</td>
<td>2</td>
<td>$x^{3} + y^{3} + axy + bx + cy$</td>
<td>$3x^{2}+ay+b+3y^{2}+ax+c$</td>
</tr>
<tr>
<td>Elliptic</td>
<td>3</td>
<td>2</td>
<td>$x^{3} + a(x^{2} + y^{2}) - 3xy + bx + cy$</td>
<td>$3x^{2}-3y^{2}+2ax+b-6xy+2ay+c$</td>
</tr>
<tr>
<td>Parabolic</td>
<td>4</td>
<td>2</td>
<td>$x^{3}y + y^{3} + ax^{2} + by + cx + dy$</td>
<td>$2xy+2ax+c+x^{2}+4y^{2}+2by+d$</td>
</tr>
</tbody>
</table>

3 - PROBLEM FORMULATION

As mentioned in the last paragraph of the introduction, the voltage collapse phenomena needs more explanation for voltage instability. The basic configuration used to explain voltage collapse is shown in figure 1-a. In this circuit the voltage source $E_{th}$ in series with the equivalent transmission impedance $Z_{th}$ represents the Thevenin equivalent of a network connected to a load. The load is described by its real and reactive powers as shown.

![Equivalent Network](image)

**Figure 1.a:** Thevenin Equivalent Network.

The load receiving end voltage $V_2$ in terms of $P_2$ and $Q_2$ and the $Z_{th}$ is obtained as follows [17]:

$$V_2 = E_{th} - \frac{S_2^*Z_{th}}{V_2^*} \tag{4}$$

Therefore,

$$E_{th}V_2^* = V_2V_2^* + S_2^*Z_{th} \tag{5}$$
Where,
\[ V_2 = |V_2| \angle \alpha, \quad E_{th} = |E_{th}| \angle 0, \quad S_2 = P_2 + j Q_2 \quad \text{and} \quad Z_{th} = R_{th} + j X_{th} \]

Rewrite eqn.(5) in terms of real and imaginary components:
\[
\begin{align*}
V_2 E_{th} \cos \alpha &= V_2^2 \left( P_2 R_{th} + Q_2 X_{th} \right) \\
V_2 E_{th} \sin \alpha &= - \left( P_2 X_{th} - Q_2 R_{th} \right)
\end{align*}
\]

Equations (6) represent the PV relationship and figure 1-b shows this for 2-bus network. For a constant power demand, there exists two operating points except the saddle point which represents the steady state stability limit. The higher operating voltage is called the stable operating point and is the feasible solution. On the contrary, the lower one is the unstable point and is infeasible [18]. In this paper, it will be shown that the higher operating point has positive value of the new proposed indicator using Catastrophe theory, the Jacobian matrix determinant, and the Eigenvalue of the reduced Jacobian matrix and lower one is negative for them. The determinant approaches zero if the operating move to the saddle point.

\[ \text{Figure 1-b: Receiving Power vs. Load Voltage for two-node Network.} \]

Since short circuit capability equals \[ \text{Scc} = \frac{E_{th}^2}{Z_{th}} \], then dividing both sides of equations (6) by \( \text{Scc} \), we obtain the following equations:
\[
\begin{align*}
f &= - (P X_{th} - Q R_{th}) / Z_{th} \\
\text{and} \\
e &= V^2 + (P R_{th} + Q X_{th})
\end{align*}
\]
where,

\[ e = \frac{V_e \cos \alpha}{E_{th}}, \quad f = \frac{V_e \sin \alpha}{E_{th}}, \quad P = \frac{P_{SCC}}{SCC}, Q = \frac{Q_{SCC}}{SCC}, \text{and} \quad V = e + jf \]

Squaring and adding eqns (7), we obtain the following equation:

\[ V^4 + V^2 \left( \frac{2(P_{R_{th}} + Q_{X_{th}})}{Z_{th}} - 1 \right) + (P^2 + Q^2) = 0 \tag{8} \]

or,

\[ V^4 + aV^2 + bV + c = 0 \tag{9} \]

where,

\[ a = \frac{2(P_{R_{th}} + Q_{X_{th}})}{Z_{th}} - 1 \]
\[ b = 0 \]
\[ c = (P^2 + Q^2) \]

Equation (9) is seen to be the swallowtail catastrophe manifold, with load bus voltage as a state variable and control variables \( a, b = 0, \) and \( c \) represent the system parameters. As seen the control variables depend upon the load active and reactive components as well as the system equivalent Thevenin impedance. Eqn (9) has the solution in \( V^2 \) as:

\[ V^2 = \left( \frac{-a}{2} \pm \sqrt{(a/2)^2 - c} \right) \tag{10} \]

### 3.1 The Critical Voltage of Loading Node

Next, we find the singularity set \( S \) which is the subset of the catastrophe manifold equation (9) that consists of all singular points corresponding to the system critical voltage stability. These are the points at which the first derivative of equation (9) equal to zero, as follows:

\[ 4V^3 + 2aV = 0 \]

or

\[ V_{cr} = \pm \sqrt[3]{\frac{-a}{2}} \tag{11} \]

It is clear that for stable operation, the control parameter \( a \) should be less than zero, and

\[ 0 < c < \left( \frac{a}{2} \right)^2 \]

to get real solution and is considered here a new indicator for voltage stability. The indicator can now be determined for each load bus in the power system. The weakest bus in the system will have the smallest magnitude for the indicator \( c \) as well as the smallest eigenvalue of the Jacobian \( J_x \) indicating that the system is on the verge of instability.
The singularity set \( S \) is then projected down onto a two-dimensional control space \((a,c)\) to obtain the bifurcation set \( \beta \). The bifurcation set is the image of catastrophe manifold in the control space which provides the region of all possible stable voltage operations in terms of the control variables \((a, c)\), which usually represent the system parameters as illustrated in figure 2.

By substituting equation (7) at critical operation into equation (11), we obtain the value of critical voltage:

\[
V_{cr} = \frac{E_{th}}{2 \cos \alpha}
\] (12)

### 3.2 The Maximum Power Loading

Corresponding to maximum loading, there is one value of \( V_{cr} \) as the two values are superimposed, then the term under the root of equation (10) must vanishes, i.e, the term under the root becomes zero, then,

\[
c = \left( \frac{a}{2} \right)^2
\]

or

\[
P^2 + Q^2 = \left( \frac{PR_{th} + QX_{th}}{Z_{th}} - \frac{1}{2} \right)^2
\] (13)
Equation (13) can be rewritten in the general form as:

\[ LP^2 + KP + M = 0 \]  \hspace{1cm} (14)

P in equation (14) is \( P_{\text{max}} \) of the node under study. Solution of equation (14) results in two values of P, one of them is negative and the other is positive. Evidently the positive value should be considered. Therefore,

\[ P_{\text{max}} = \left( \frac{1}{2L} \right) (-K \pm \sqrt{K^2 - 4LM}) \]  \hspace{1cm} (15)

where,

\[ L = 1 \]
\[ K = -\frac{R_{th}(2QX_{th} - Z_{th})}{X^2} \]
\[ M = \frac{(R_{th}Q)^2}{X_{th}} + \frac{QZ_{th}}{X_{th}} - \left( \frac{Z_{th}}{2X_{th}} \right)^2 \]

As seen the presence of a reactive component of load Q and system impedance affects the loading power limit.

4 - VOLTAGE STABILITY EVALUATION USING JACOBIAN MATRIX ELEMENTS

The use of the singularity of power flow Jacobian matrix as an indicator of steady state stability was first pointed out by reference [19], where the sign of the determinant of \( J_x \) was used to determine if the studied operating point was stable or not.

From energy conservation, the following active and reactive load flow equations for P and Q can be obtained from equations (7) and taken negative (injected at load bus):

\[ -P = \frac{R_{th}}{Z_{th}} (e - V^2) - \frac{X_{th} \cdot f}{Z_{th}} \]  \hspace{1cm} (16)
\[ -Q = \frac{X_{th}}{Z_{th}} (e - V^2) + \frac{R_{th} \cdot f}{Z_{th}} \]  \hspace{1cm} (17)

From the linearized Power flow equations (16) and (17) we obtain

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial P}{\partial f} & \frac{\partial P}{\partial e} \\
\frac{\partial Q}{\partial f} & \frac{\partial Q}{\partial e}
\end{bmatrix}
\begin{bmatrix}
\Delta f \\
\Delta e
\end{bmatrix}
\]  \hspace{1cm} (18)

where, the elements of the Jacobian matrix of the system is
\[ \frac{\partial P}{\partial f} = \frac{X_{th} + 2R_{th}f}{Z_{th}}, \quad \frac{\partial P}{\partial e} = \frac{R_{th}(2e-1)}{Z_{th}} \]

\[ \frac{\partial Q}{\partial f} = \frac{2X_{th}f - R_{th}}{Z_{th}}, \quad \frac{\partial Q}{\partial e} = \frac{X_{th}(2e-1)}{Z_{th}} \]

and the determinant of the Jacobian matrix is

\[ |J_x| = Z_{th}(2e - 1) \quad (19) \]

The singular points corresponding to the system critical voltage stability occur when the determinant of the Jacobian matrix is zero. Then at \( |J_x| = 0 \), we obtain

\[ e_{cr} = \frac{1}{2} \]

or

\[ V_{cr} = \frac{E_{th}}{2 \cos \alpha} \quad (21) \]

Equation (21) represents the critical load bus voltage and as seen the same equation obtained using the catastrophe theory (equation (12)). After a little manipulation for equations (16) and (17) we have:

\[ (e - \frac{1}{2})^2 + (f + \frac{X_{th}}{2R_{th}})^2 = \left( \frac{Z_{th}^2 - 4R_{th}Z_{th}P}{4R^2} \right) \]

and

\[ (e - \frac{1}{2})^2 + (f + \frac{R_{th}}{2X_{th}})^2 = \left( \frac{Z_{th}^2 - 4X_{th}Z_{th}Q}{4X^2} \right) \]

where, the value of \( f \) is given in equation (7) and the value of \( e \) is recalculated and given by:

\[ e = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{Z_{th}^2} [(X_{th}P - R_{th}Q)^2 + Z_{th}(R_{th}P + X_{th}Q)]} \quad (23) \]

Equations (22) define two circles on \((e,f)\) plane shown in Figure 3, with centers at

\[ \left( \frac{1}{2}, \frac{X_{th}}{2R_{th}} \right), \left( \frac{1}{2}, \frac{R_{th}}{2X_{th}} \right) \]

and radii of

\[ \sqrt{\frac{Z_{th}^2 - 4R_{th}Z_{th}P}{4R_{th}^2}} \quad \text{and} \quad \sqrt{\frac{Z_{th}^2 - 4X_{th}Z_{th}Q}{4X_{th}^2}} \]

respectively. The radii vary with the real and reactive load respectively. The heavier the load is, the smaller the radii will be. The intersection points define the possible values of voltages \((e,f)\) at the load bus.
From figure 3, it is clear that there are two solutions (higher and lower) for the load bus voltage \((e, f)\) as long as \(\frac{X_{th} P - Q R_{th}}{Z_{th}}\) is less than the radius of the circle. Also to get the real roots for \(e\), we have:

\[
(X_{th}P - R_{th}Q)^2 + Z_{th}(R_{th}P + X_{th}Q) \leq \left(\frac{Z_{th}}{2}\right)^2
\]  

Equation (20) represents the vertical line connects the centers of the circles defined by equations (22). This line can be considered as the border between the stable (area A right hand side) and unstable (Area B left hand side) areas in the voltage plane, which is similar to the border curve obtained at \(V_{cr} = \frac{E_{th}}{2 \cos \alpha}\) or at \(e_{cr} = \frac{1}{2}\) from the catastrophe theory.

**5 - VOLTAGE STABILITY EVALUATION USING MODAL ANALYSIS**

Voltage stability characteristics of the system can be identified by computing the eigenvalues of the reduced Newton-Raphson load flow Jacobian matrix. The linearized steady-state system power voltage equations are illustrated in equation (18) and the variation in reactive load flow \(\Delta Q\) at constant active power component (i.e \(\Delta P = 0\)) is given by:
\[ \Delta Q = (J_{Qe} - J_{Qf} J_{p_f}^{-1} J_{p_e}) \Delta e \]

or

\[ \Delta Q = J_{xR} \Delta e \]

where,

\[ J_{xR} = (J_{Qe} - J_{Qf} J_{p_f}^{-1} J_{p_e}) \]

\[ J_{pf} = (X_{th} + 2R_{th}f)/Z_{th}, \quad J_{pe} = R_{th}(2e-1)/Z_{th} \]

\[ J_{Qf} = (2X_{th}f - R_{th})/Z_{th}, \quad J_{Qe} = X_{th}(2e - 1)/Z_{th} \]

\( J_{xR} \) is a reduced Jacobian matrix of a two node system, and its dimension is 1 x 1 matrix. Eigenvalue of this reduced Jacobian matrix is used as indicator of voltage stability. If the eigenvalue of \( J_{xR} \) is positive, the system is considered voltage stable. The system is considered voltage unstable if the eigenvalue of \( J_{xR} \) is negative. At each operating point we keep the active power component constant and evaluate voltage stability by considering the incremental relationship between the reactive power \( Q \) and the magnitude of active bus voltage \( e \).

**6 – NUMERICAL EXAMPLE**

In order to study the intrinsic characteristics of the catastrophe control parameters \((a, c)\), the singularity of power flow through the determinant sign of the Jacobian matrix and the eigenvalue of the reduced Jacobian matrix using modal analysis and their relations to voltage instability state, a theoretical test system has been adopted. The test system is the Ward-Hale 6-bus Network [20] shown in Figure 4. This system, generally used for testing the new developed techniques in comparison to the known ones. The initial data for generation and load of the system is given in Table 2. The results of the conventional load flow run of the test system are shown in Table 3. In this run, bus 1 is taken as the slack bus and bus 2 is maintained at constant voltage of 1.1 p.u. An accuracy of 0.00001 was achieved in both voltage magnitude and angle.

![Figure 4: Ward-Hale 6-bus Network.](image-url)
Table 2: Bus Loading and Voltages.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Bus Voltage</th>
<th>Active Power</th>
<th>Reactive Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.050</td>
<td>……</td>
<td>……</td>
</tr>
<tr>
<td>2</td>
<td>1.100</td>
<td>0.500</td>
<td>……</td>
</tr>
<tr>
<td>3</td>
<td>……</td>
<td>-0.550</td>
<td>-0.13</td>
</tr>
<tr>
<td>4</td>
<td>……</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>5</td>
<td>……</td>
<td>-0.300</td>
<td>-0.18</td>
</tr>
<tr>
<td>6</td>
<td>……</td>
<td>-0.500</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

For this system, the Thevenin’s equivalent (E_{th} and Z_{th}) at each load node with its load separated is firstly calculated. The Thevenin’s p.u. voltages E_{th} and impedances Z_{th} for the load nodes (nodes 3 – 6) are found to be as given in Table 4.

Table 3: Load Flow Results.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Bus Voltage</th>
<th>Active Power</th>
<th>Reactive Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.050 \angle 0</td>
<td>0.952</td>
<td>0.433</td>
</tr>
<tr>
<td>2</td>
<td>1.100 \angle -3.34</td>
<td>0.500</td>
<td>0.184</td>
</tr>
<tr>
<td>3</td>
<td>1.0008 \angle -12.78</td>
<td>-0.550</td>
<td>-0.13</td>
</tr>
<tr>
<td>4</td>
<td>0.9290 \angle -9.84</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.9198 \angle -12.33</td>
<td>-0.300</td>
<td>-0.18</td>
</tr>
<tr>
<td>6</td>
<td>0.9192 \angle -12.24</td>
<td>-0.500</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Table 4: Thevenin’s equivalents.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Thevenin’s Voltage</th>
<th>Thevenin’s impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.975 \angle -1.023</td>
<td>0.2740 \angle 75.64</td>
</tr>
<tr>
<td>4</td>
<td>0.954 \angle -9.93</td>
<td>0.2220 \angle 75.81</td>
</tr>
<tr>
<td>5</td>
<td>1.000 \angle -3.1987</td>
<td>0.32315 \angle 75.43</td>
</tr>
<tr>
<td>6</td>
<td>0.987 \angle -3.21</td>
<td>0.2370 \angle 75.54</td>
</tr>
</tbody>
</table>

The new indicator c , the singular value of the Jacobian |J_x| and the eigenvalues λ using modal analysis are applied and calculated for each load node at the nominal load setting. In order to test these indicators, two tests are carried out:

6.1 The First Test
Stressing each load node of the test Ward-Hale 6-bus Network with keeping the reactive power constant at each load node by gradually increases the active power, starting from the base load up to 130% P_{\text{max}}. The results show complete agreement, also, it shows the value of the critical voltage V_{cr} for each load node and its corresponding consumed max power P_{\text{max}}. The criteria for the system to be stable, the new indicator c should be less than (a/2)^2 , the determinant values of the Jacobian |J_x| and the eigenvalues λ also should be positive values as shown in Table 5. The new
indicator c using the catastrophe theory provides a way to determine the stable and unstable zones as shown in figure 5. As seen the break away point which represents the critical loading point at each bus occurs at loading with maximum value. This point corresponding to the saddle point obtained from P-V relationship as shown in figure 1-b which separates between the stable and unstable zones.

**Table 5-a**: The New and Different Voltage Stability Indicators for Load Bus 3 for 6 – Bus Power System.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>New indicator c</th>
<th>Jacobian Value</th>
<th>Eigenvalue λ</th>
<th>Critical Voltage V_c</th>
<th>Bus Loading</th>
<th>System State</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS 3</td>
<td>0.0265 -0.8319</td>
<td>0.2097</td>
<td>3.3550</td>
<td>0.629</td>
<td>Base Load</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.0327 -0.8205</td>
<td>0.2018</td>
<td>3.2540</td>
<td>0.625</td>
<td>50% P_max</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.0465 -0.7987</td>
<td>0.1842</td>
<td>3.0559</td>
<td>0.616</td>
<td>60% P_max</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.0719 -0.7663</td>
<td>0.1500</td>
<td>2.6074</td>
<td>0.604</td>
<td>75% P_max</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.103 -0.7340</td>
<td>0.9751</td>
<td>1.8067</td>
<td>0.591</td>
<td>90% P_max</td>
<td>Stable</td>
</tr>
<tr>
<td>Maximum Loading p.u 1.2289</td>
<td>0.1268 -0.7122</td>
<td>0.0000</td>
<td>0.1219</td>
<td>0.582</td>
<td>100% P_max</td>
<td>Critical Stable</td>
</tr>
<tr>
<td></td>
<td>0.1532 -0.6908</td>
<td>-0.41100</td>
<td>-7.403087</td>
<td>0.573</td>
<td>110% P_max</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.1820 -0.6690</td>
<td>-0.41100</td>
<td>-7.624514</td>
<td>0.564</td>
<td>120% P_max</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>0.2134 -0.6475</td>
<td>-0.41100</td>
<td>-7.859595</td>
<td>0.555</td>
<td>130% P_max</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

**Figure 5-a**: Operating Zones for Bus 3.
Table 5-b: The New and Different Voltage Stability Indicators for Load Bus 4 for 6 – Bus Power System.

| Bus Number | New indicator | Jacobian Value $|J_x|$ | Eigenvalue $\lambda$ | Critical Voltage $V_{cr}$ | Bus Loading | System State |
|------------|---------------|-----------------|------------------------|--------------------------|-------------|--------------|
| BUS 4      |               |                 |                        |                          |             |              |
| Maximum Loading p.u 1.573 | | | | | | |
| 0.0000     | -1.000        | 0.222           | 4.943280               | 0.674580                 | Base Load   | Stable       |
| 0.0368     | -0.900        | 0.17595         | 4.531556               | 0.634206                 | 50% $P_{max}$ | Stable       |
| 0.0530     | -0.860        | 0.16139         | 4.277920               | 0.625819                 | 60% $P_{max}$ | Stable       |
| 0.0828     | -0.826        | 0.13414         | 3.688300               | 0.613023                 | 75% $P_{max}$ | Stable       |
| 0.1193     | -0.791        | 0.85462         | 2.608000               | 0.599955                 | 90% $P_{max}$ | Stable       |
| 0.1473     | -0.768        | 0.00000         | 0.287339               | 0.591082                 | 100% $P_{max}$ | Critical Stable |
| 0.1783     | -0.744        | -0.333000       | -9.384356              | 0.582073                 | 110% $P_{max}$ | Unstable     |
| 0.2122     | -0.721        | -0.33300        | -9.707075              | 0.572923                 | 120% $P_{max}$ | Unstable     |
| 0.2490     | -0.698        | -0.33300        | -10.05278              | 0.563625                 | 130% $P_{max}$ | Unstable     |

Figure 5-b: Operating Zones for Bus 4.
Table 5-c: The New and Different Voltage Stability Indicators for Load Bus 5 for 6 – Bus Power System.

| Bus Number | New indicator | Jacobian Value $|J_x|$ | Eigenvalue $\lambda$ | Critical Voltage $V_{cr}$ | Bus Loading | System State |
|------------|---------------|-----------------|-----------------|-------------------|-------------|--------------|
| BUS 4      | 0.0104        | -0.8787         | 0.276166        | 2.936918          | 0.663       | Base Load    | Stable       |
| Max Loading p.u 1.0398 | 0.0316        | -0.7856         | 0.226402        | 2.510242          | 0.627       | 50% $P_{max}$ | Stable       |
|            | 0.0440        | -0.7647         | 0.206671        | 2.345026          | 0.618       | 60% $P_{max}$ | Stable       |
|            | 0.0668        | -0.7337         | 0.168214        | 1.977991          | 0.606       | 75% $P_{max}$ | Stable       |
|            | 0.0948        | -0.7025         | 0.109354        | 1.334347          | 0.593       | 90% $P_{max}$ | Stable       |
|            | 0.1162        | -0.6817         | 0.000000        | 0.000000          | 0.584       | 100% $P_{max}$| Critical Stable |
|            | 0.1399        | -0.6612         | -0.161575       | -2.013231         | 0.575       | 110% $P_{max}$| Unstable      |
|            | 0.1659        | -0.6403         | -0.161575       | -2.066252         | 0.566       | 120% $P_{max}$| Unstable      |
|            | 0.1941        | -0.6197         | -0.161575       | -2.122141         | 0.557       | 130% $P_{max}$| Unstable      |

Figure 5-c: Operating Zones for Bus 5.
**Table 5-d:** The New and Different Voltage Stability Indicators for Load Bus 6 for 6 – Bus Power System.

| Bus Number       | New indicator | Jacobian Value $| J_x | | Eigenvalue $\lambda$ | Critical Voltage $V_{cr}$ | Bus Loading | System State |
|------------------|---------------|-----------------|-----|------------------------|--------------------------|-------------|--------------|
| BUS 6            | 0.0149        | -0.902          | 0.205823 | 4.216143 | 0.663 | Base Load | Stable   |
|                  | 0.0350        | -0.862          | 0.184078 | 3.949922 | 0.648 | 50% $P_{max}$ | Stable   |
|                  | 0.0503        | -0.839          | 0.168060 | 3.709058 | 0.639 | 60% $P_{max}$ | Stable   |
|                  | 0.0786        | -0.805          | 0.136814 | 3.159393 | 0.626 | 75% $P_{max}$ | Stable   |
| Maximum Loading p.u 1.5357 | 0.1132 | -0.770          | 0.088957 | 2.169858 | 0.613 | 90% $P_{max}$ | Stable   |
|                  | 0.1397        | -0.748          | 0.000000 | 0.074827 | 0.603 | 100% $P_{max}$ | Critical Stable   |
|                  | 0.1690        | -0.724          | -0.35550 | -8.825145 | 0.594 | 110% $P_{max}$ | Unstable   |
|                  | 0.2011        | -0.702          | -0.35550 | -9.108338 | 0.585 | 120% $P_{max}$ | Unstable   |
|                  | 0.2360        | -0.679          | -0.35550 | -9.410307 | 0.575 | 130% $P_{max}$ | Unstable   |

**Figure 5-d:** Operating Zones for Bus 6.

### 6.2 The Second Test

As illustrated in Table 5-c the values of the control parameter $c$ of bus 5 as compared to the other buses are the smallest values as well as its eigenvalues obtained. Therefore bus 5 is considered here the weakest bus in the system. In order to test the three criteria
for this bus with load active power is kept constant at 0.3 p.u and load reactive powers are assumed to be increased gradually from 0.18 p.u up to 402.8 % of its nominal value i.e reactive power of that bus is increased to 0.725 p.u. The node critical voltages nearly dropped down by 25 % from stable to unstable cases.

The criteria for the system to be stable, the new indicator c should be less than \((a/2)^2\), the determinant values of the Jacobian \(|J_x|\) and the eigenvalues \(\lambda\) also should be positive values as shown in Table 6. As shown in figure 6 the break away point which represents the critical reactive power loading point at bus 5 occurs at 0.715 p.u. This agree with the given results.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>New indicator</th>
<th>Jacobian Value</th>
<th>Eigenvalue</th>
<th>Critical Voltage Vcr</th>
<th>Bus 5 VARs p.u.</th>
<th>System State</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS 5</td>
<td>c a</td>
<td>(</td>
<td>J_x</td>
<td>)</td>
<td>(\lambda)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01040 -0.87865</td>
<td>0.276166</td>
<td>2.936918</td>
<td>0.6630</td>
<td>0.18</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.02229 -0.71861</td>
<td>0.210677</td>
<td>2.201371</td>
<td>0.6000</td>
<td>0.36</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.03890 -0.60795</td>
<td>0.148268</td>
<td>1.530762</td>
<td>0.5510</td>
<td>0.54</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.05080 -0.44281</td>
<td>0.103353</td>
<td>1.060713</td>
<td>0.5260</td>
<td>0.63</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.06200 -0.50359</td>
<td>0.024072</td>
<td>0.245756</td>
<td>0.5020</td>
<td>0.710</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.06230 -0.50239</td>
<td>0.018062</td>
<td>0.184371</td>
<td>0.5010</td>
<td>0.712</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.06240 -0.50159</td>
<td>0.014128</td>
<td>0.144204</td>
<td>0.5009</td>
<td>0.713</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.06260 -0.50119</td>
<td>0.008541</td>
<td>0.087175</td>
<td>0.5006</td>
<td>0.714</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>0.62700 -0.50040</td>
<td>-0.161575</td>
<td>-1.501579</td>
<td>0.5003</td>
<td>0.715</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>0.63500 -0.49719</td>
<td>-0.161575</td>
<td>-1.500193</td>
<td>0.4988</td>
<td>0.720</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>0.64200 -0.49437</td>
<td>-0.161575</td>
<td>-1.498810</td>
<td>0.4972</td>
<td>0.725</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

**Table 6:** The New and Different Voltage Stability Indicators for Bus 5 at different Loading VARs With Constant Active Power 0.3 p.u.

![Stress Bus 5 With Different VARs at Constant Active Power 0.3 p.u.](image)

**Figure 6:** Operating Zones for Bus 5 at Different VARs With Constant active Power.
7- CONCLUSIONS

1. This paper has developed a new voltage stability indicator for integrated power systems using the catastrophe theory. A swallowtail manifold has been shown to be appropriate for this new indicator.

2. An algorithm has been presented in this paper and a developed C – language program is used to calculate the new indicator in terms of the system parameters; load active and reactive powers, transmission impedance, and the voltage source. These system parameters can be measured or calculated with a speed adequate for on-line steady-state voltage stability assessment. It has been shown that the zone inside the border curve which represents the curve of critical points and the curve of bus operating points represents the steady voltage stable zone, and the voltage collapse occurs in the zone that produced after the system is loaded with value greater than maximum load \( P_{\text{max}} \).

3. Coincidence of the obtained results of the new indicator, determinant of the Jacobian, and the eigenvalues is proved for system load nodes of different active and reactive power loadability conditions leading to stable and unstable voltage cases.

8 – REFERENCES

A NEW VOLTAGE COLLAPSE PROXIMITY INDICATOR


Mothers the exacerbation of new disappearing voltage

This research registers a new method of evaluating voltage instability in power systems by comparing the traditional method used in the vicinity. This method is characterized by the calculation of the Jacobian determinant for electrical power systems and the modal analysis. These methods are used to define the region of the voltage collapse in the operation of the system. The results obtained from this study agree with the existing phenomena, indicating that the new indicator of the voltage collapse has occurred in the region of the balanced operation.

This research introduced a new indicator for detecting voltage collapse in power systems. The indicator is based on the sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters. The indicator is calculated using the Jacobian determinant of the power system and the modal analysis. The results obtained from this study agree with the existing phenomena, indicating that the new indicator of the voltage collapse has occurred in the region of the balanced operation.

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The results of this study can be applied to a number of cases in the field of power systems, including the Ward-Hale 6-bus network and other similar systems. The indicator provides a clear and accurate indication of the critical points in the voltage collapse, which is crucial for the operation and planning of power systems.

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تكون قيمة $c$ موجبة وأقل من $(a/2)^2$ وهو ما يتطابق مع قيمة محدد مصفوفة سريان Eigenvalues.

تم إجراء اختباران على الشبكة المستخدمة: الاختبار الأول تم فيه تحميل قضبان الأحمال المختلفة بقيم للقدرة الفعالة تصل إلى 130% من الحمل الأقصى الذي تم استنتاجه خلال هذا البحث مع فرض مركبة القدرة الغير فعالة لهذه الأحمال ثابتة للناتج، حيث كانت النتائج التي تم الحصول عليها تتطابق مع الطرق التقليدية الأخرى.

أما الاختبار الثاني فقد تم فيه الدراسة على القضبان الواهنة التي تم تحديدها أيضا في هذا البحث وتحملها بأحمال ذات قدرة فعالة ثابتة القيمة وبقدرة غير فعالة متغيرة القيمة، أيضاً أمكن تحديد منطقة التشغيل المتزنة وكذلك النقاط الفاصلة التي بعدها يحدث الانهيار في جهد تشغيل النظام مما يعطي لهذا المؤشر الجديد إمكانية التطبيق العملي والمراقبة الفورية لمناطق تشغيل قضبان التحميل بشبكات القوى الكهربية.