INCORPORATING COMBINED FACTS IN LOAD FLOW STUDIES

M. Z. EL-Sadek, A. Ahmed
Electrical Engineering Department, Faculty of Engineering, Assiut University Assiut, Egypt

M. A. Mohammed
Country Electricity County, Souhag, Egypt

(Received May 3, 2008 Accepted May 28, 2008)

With the restructuring of electric power industry, there has been great interest in the combined FACTS. In this paper, mathematical models of FACTS controllers such as the STATCOM, SSSC, UPFC and the latest IPFC and GUPFC are established and a nonlinear optimization framework with comprehensive modeling of these FACTS controllers is proposed. Numerical examples based on the IEEE 30 bus system are given to show the effectiveness of the mathematical models for the combined FACTS controllers.

KEYWORDS: Flexible AC transmission system (FACTS), Static Synchronous Compensator (STATCOM), Static Synchronous Series Compensator (SSSC), Unified Power Flow Controller (UPFC), Interline Power Flow Controller (IPFC), Generalized Unified Power Flow Controller (GUPFC), Static VAR Compensator (SVC), Newton power flow, MATLAB.

1. INTRODUCTION

Several publications have recently appeared in FACTS literature which describe the basic operating principles of FACTS [1-4]. Very little work has been done in developing suitable models for assessing FACTS behaviors in large-scale power networks.

Fortunately, new technologies are becoming available that will help electricity companies maintain power system reliability while handling large volumes of transactions, among which FACTS can support system voltage at specified buses, control power flow on transmission networks and maximize use of the exciting transmission assets.

Greater reliability in electrical power delivery is now a necessity for both industrial and individual utility customers. Therefore, it is expected that application of FACTS controllers will be growing, particularly in the deregulated electricity market environment.

In recent years there are converter based FACTS controllers available, these FACTS have much stronger voltage control capabilities than that of SVC; furthermore these except the shunt FACTS controllers have power flow control capabilities. In the converter based FACTS controllers family, there are single function controllers such as the STATCOM [5], SSSC [4], and also multi-functional and multi-converter FACTS...
controllers such as the UPFC [3], IPFC and GUPFC [6]. Incorporating combined FACTS in load flow studies is the main concern of this paper. The study achieved results which show the advantages of using such combined FACTS in power systems applications.

2. COMPREHENSIVE MATHEMATICAL MODELING OF FACTS CONTROLLERS

2.1 STATCOM [5]

A STATCOM is usually used to control transmission voltage by reactive power compensation. Typically, a STATCOM consists of a coupling transformer, an inverter and a DC capacitor, which is shown in Fig. (1). For such an arrangement, in ideal steady state analysis, it can be assumed that the active power exchange between the AC system and the STATCOM can be neglected, and only the reactive power can be exchanged between them. The equivalent circuit of a STATCOM is given in Fig. (2).

![Fig. (1): STATCOM components.](image1)

![Fig. (2): STATCOM equivalent circuit.](image2)

In principle, the STATCOM output voltage can be regulated such that the reactive power of the STATCOM can be changed. According to the equivalent circuit of the STATCOM shown in Fig. (2) the power flow constraints of the STATCOM are:

\[
P_{sh} = V_i^2 g_{sh} - V_i V_{sh} (g_{sh} \cos(\theta_i - \theta_{sh}) + b_{sh} \sin(\theta_i - \theta_{sh}))
\]

\[
Q_{sh} = -V_i^2 b_{sh} - V_i V_{sh} (g_{sh} \sin \cos(\theta_i - \theta_{sh}) - b_{sh} \sin(\theta_i - \theta_{sh}))
\]

Where \( g_{sh} + j b_{sh} = 1/Z_{sh} \).

Operating constraint of the STATCOM – the active exchange via the DC link is zero, which is described by

\[
PE = \text{Re} (V_{sh} I_{sh}^*) = 0
\]

Where \( \text{Re} (V_{sh} I_{sh}^*) = V_{sh}^2 g_{sh} - V_i V_{sh} (g_{sh} \cos(\theta_i - \theta_{sh}) - b_{sh} \sin(\theta_i - \theta_{sh})) \).
The bus voltage control as follows,

\[ V_i - V_i^{\text{Spec}} \]

Where \( V_i^{\text{Spec}} \) is the specified bus voltage.

The equivalent voltage injection \( V_{sh} \) and its phase angle \( \theta_{sh} \) bound constraints:

\[ V_{sh}^{\text{min}} \leq V_{sh} \leq V_{sh}^{\text{max}} \]
\[ -\pi \leq \theta_{sh} \leq \pi \quad (4) \]

Where \( V_{sh}^{\text{max}} \) is the voltage rating of the STATCOM, while \( V_{sh}^{\text{min}} \) is the minimal voltage limit of the STATCOM.

### 2.2 SSSC [4]

A SSSC usually consists of a coupling transformer, an inverter and a capacitor as shown in Fig. (3). However, a SSSC is series connected with a transmission line through the coupling transformer. In principle, the inserted series voltage \( V_{ser} \) can be regulated to change the impedance of the transmission line. Therefore the power flow of the transmission line can be controlled.

![Fig. (3): SSSC operation principle.](image)

![Fig. (4): SSSC equivalent circuit.](image)

The equivalent circuit of a SSSC as shown in Fig. (4) can be derived according to the its operation principle shown in Fig. (3). According to the equivalent circuit, the power flow equations of the SSSC can be established:

\[ P_{ij} = V_i^2 g_{ii} - V_i V_j (g_{ij} \cos (\theta_i - \theta_j) + b_{ij} \sin (\theta_i - \theta_j)) - V_i V_{ser} (g_{ij} \cos (\theta_i - \theta_{ser}) + b_{ij} \sin (\theta_i - \theta_{ser})) \quad (6) \]

\[ Q_{ij} = -V_i^2 b_{ii} - V_i V_j (g_{ij} \sin (\theta_i - \theta_j) - b_{ij} \cos (\theta_i - \theta_j)) - V_i V_{ser} (g_{ij} \sin (\theta_i - \theta_{ser}) - b_{ij} \cos (\theta_i - \theta_{ser})) \quad (7) \]

\[ P_{ji} = V_j^2 g_{jj} - V_i V_j (g_{ij} \cos (\theta_j - \theta_i) + b_{ij} \sin (\theta_j - \theta_i)) + V_j V_{ser} (g_{ij} \cos (\theta_j - \theta_{ser}) + b_{ij} \sin (\theta_j - \theta_{ser})) \quad (8) \]

\[ Q_{ji} = -V_j^2 b_{jj} - V_i V_j (g_{ij} \sin (\theta_j - \theta_i) - b_{ij} \cos (\theta_j - \theta_i)) + V_j V_{ser} (g_{ij} \sin (\theta_j - \theta_{ser}) - b_{ij} \cos (\theta_j - \theta_{ser})) \quad (9) \]

Where \( g_{ij} + j b_{ij} = 1 / Z_{ser} \), \( g_{ii} = g_{ij} \), \( b_{ii} = b_{ij} \), \( g_{jj} = g_{ij} \), \( b_{jj} = b_{ij} \).

The operating constraint of the SSSC (the active power exchange via the dc link) is
\[ PE = \text{Re} \left( V_{\text{ser}}^* I_{\text{ser}}^* \right) = 0 \]  

\[ \text{Re} \left( V_{\text{ser}}^* I_{\text{ser}}^* \right) = -V_i V_{\text{ser}} (g_{ij} \cos (\theta_i - \theta_{\text{ser}}) - b_{ij} \sin (\theta_i - \theta_{\text{ser}})) + V_j V_{\text{ser}} (g_{ij} \sin (\theta_j - \theta_{\text{ser}}) - b_{ij} \cos (\theta_j - \theta_{\text{ser}})) \]  

The mathematical descriptions of the two control modes of the SSSC are presented as follows:

**Mode 1: Active Power Flow Control**

The active power flow control constraint is as follows:

\[ P_{ji} - P_{ji}^{\text{Spec}} = 0 \]  

**Mode 2: Reactive Power Flow Control**

The reactive power flow control constraint is as follows:

\[ Q_{ji} - Q_{ji}^{\text{Spec}} = 0 \]

\( P_{ji}^{\text{Spec}} \) and \( Q_{ji}^{\text{Spec}} \) are the specified line active and line reactive power flow.

The equivalent series voltage injection \( V_{\text{ser}} \) and its phase angle \( \theta_{\text{ser}} \) bound constraints:

\[ V_{\text{ser}}^{\text{min}} \leq V_{\text{ser}} \leq V_{\text{ser}}^{\text{max}} \]

\[ -\pi \leq \theta_{\text{ser}} \leq \pi \]

Where \( V_{\text{ser}}^{\text{max}} \) is the voltage rating of the SSSC, while \( V_{\text{ser}}^{\text{min}} \) is the minimal voltage limit of the SSSC.

### 2.3 UPFC [3]

Fig. (5) shows the basic circuit arrangement of the UPFC where it consists of two switching converters. These converters are operated from a common DC link provided by a DC storage capacitor.

According to the equivalent circuit shown in Fig. (6), then the power flow equations of the UPFC can be established:

\[ P_{ij} = V_i^2 g_{ii} - V_i V_j (g_{ij} \cos (\theta_i - \theta_j) + b_{ij} \sin (\theta_i - \theta_j)) - V_j V_{\text{ser}} (g_{ij} \cos (\theta_j - \theta_{\text{ser}}) + b_{ij} \sin (\theta_j - \theta_{\text{ser}})) - V_i V_{\text{sh}} (g_{sh} \cos (\theta_i - \theta_{\text{sh}}) + b_{sh} \sin (\theta_i - \theta_{\text{sh}})) \]
\[
\begin{align*}
Q_{ij} &= -V_i^2 b_{ii} - V_i V_j (g_{ij} \sin (\theta_i - \theta_j) - b_{ij} \cos (\theta_i - \theta_j)) \\
&\quad - V_i V_{ser} (g_{ij} \sin (\theta_i - \theta_{ser}) - b_{ij} \cos (\theta_i - \theta_{ser})) \\
&\quad - V_i V_{sh} (g_{ij} \sin (\theta_i - \theta_{sh}) - b_{ij} \cos (\theta_i - \theta_{sh})) \\
P_{ji} &= V_i^2 g_{jj} - V_i V_j (g_{ij} \cos (\theta_j - \theta_i) + b_{ij} \sin (\theta_j - \theta_i)) \\
&\quad + V_j V_{ser} (g_{ij} \cos (\theta_j - \theta_{ser}) + b_{ij} \sin (\theta_j - \theta_{ser})) \\
Q_{ji} &= -V_i^2 b_{jj} - V_i V_j (g_{ij} \sin (\theta_j - \theta_i) - b_{ij} \cos (\theta_j - \theta_i)) \\
&\quad + V_j V_{ser} (g_{ij} \sin (\theta_j - \theta_{ser}) - b_{ij} \cos (\theta_j - \theta_{ser}))
\end{align*}
\]

Where \( g_{ij} + j b_{ij} = 1 / Z_{ser} \), \( g_{ii} = g_{ij} + g_{sh} \), \( b_{ii} = b_{ij} + b_{sh} \), \( g_{jj} = g_{ij} \), \( b_{jj} = b_{ij} \).

The operating constraint of the UPFC (the active power exchange via the dc link) is

\[
PE = P_{sh} + P_{ser} = 0
\]

Where

\[
\begin{align*}
P_{sh} &= \text{Re} \left( V_{sh} I_{ji}^* \right) \\
P_{ser} &= \text{Re} \left( -V_{ser} I_{ji}^* \right)
\end{align*}
\]

The bus voltage, the active power flow and the reactive power flow control as follows,

\[
\begin{align*}
V_i - V_i^{\text{Spec}} \\
P_{ji} - P_{ji}^{\text{Spec}} \\
Q_{ji} - Q_{ji}^{\text{Spec}}
\end{align*}
\]

Where \( V_i^{\text{Spec}} \), \( P_{ji}^{\text{Spec}} \) and \( Q_{ji}^{\text{Spec}} \) are the specified bus voltage, line active and line reactive power flow.

### 2.4 IPFC [6]

An IPFC [6] with combing two or more series–connected converters working together extends the concepts of voltage and power flow control beyond what is achievable with a known one-converter FACTS controller – SSSC [4] which is shown in Fig. (7) is used to show the basic operation principle for the sake of simplicity. It can control total three power system quantities such as three independent power flow of two lines. The equivalent circuit of the IPFC consisting of two controllable series injected voltage sources is shown in Fig. (8).

![Fig. (7): IPFC operational principle.](image1)

![Fig. (8): IPFC equivalent circuit.](image2)
According to the equivalent circuit of the IPFC shown in Fig. (8), the power flow equations can be derived as:

\[ P_i = V_i^2 g_{ii} - \sum_n V_i V_n (g_{in} \cos (\theta_i - \theta_n) + b_{in} \sin (\theta_i - \theta_n)) \]
\[ + \sum_n V_i V_{serin} (g_{in} \cos (\theta_i - \theta_{serin}) + b_{in} \sin (\theta_i - \theta_{serin})) \]  
(23)

\[ Q_i = -V_i^2 b_{ii} - \sum_n V_i V_n (g_{in} \sin (\theta_i - \theta_n) - b_{in} \cos (\theta_i - \theta_n)) \]
\[ + \sum_n V_i V_{serin} (g_{in} \sin (\theta_i - \theta_{serin}) - b_{in} \cos (\theta_i - \theta_{serin})) \]  
(24)

\[ P_{ni} = V_i^2 g_{nn} - V_i V_j (g_{in} \cos (\theta_n - \theta_i) + b_{in} \sin (\theta_n - \theta_i)) \]
\[ + V_j V_{serin} (g_{in} \cos (\theta_j - \theta_{serin}) + b_{in} \sin (\theta_j - \theta_{serin})) \]  
(25)

\[ Q_{ni} = -V_i^2 b_{nn} - V_i V_j (g_{in} \sin (\theta_n - \theta_i) - b_{in} \cos (\theta_n - \theta_i)) \]
\[ + V_j V_{serin} (g_{in} \sin (\theta_j - \theta_{serin}) - b_{in} \cos (\theta_j - \theta_{serin})) \]  
(26)

Where

\[ g_{in} + j b_{in} = 1 / Z_{serin} , \ g_{ii} = \sum_n g_{in} , \ b_{ii} = \sum_n b_{in} , \ g_{nn} + j b_{nn} = 1 / Z_{serin} (n = j,k,...) . \]

The equivalent controllable injected voltage source magnitude and angle of the series converter are constrained by

\[ V_{serin}^{\min} \leq V_{ser} \leq V_{serin}^{\max} \]  
(27)

\[ -\pi \leq \theta_{serin} \leq \pi \]  
(28)

Where \( n = j, k \), \( V_{serin}^{\max} \), \( V_{serin}^{\min} \) are the maximal and minimal voltage limits of \( V_{serin} \) respectively. According to the operating principle of the IPFC, the operating constraint representing the active power exchange between or among the converters via the common DC link is given by

\[ PE = \sum_n P_{serin} = 0 \ (n = j,k,...) \]  
(29)

Where

\[ P_{serin} = \text{Re} \ (V_{serin} I_{serin}) \ (n = j,k,...) \]  
(30)

Where \( I_{serin}^* \) is conjugate of \( I_{serin} \).

The active and the reactive power flow control constraints of the IPFC shown in Fig. (8) are:

\[ P_{ji} - P_{ji}^{\text{Spec}} = 0 \]  
(31)

\[ Q_{ji} - Q_{ji}^{\text{Spec}} = 0 \]  
(32)

\[ P_{ki} - P_{ki}^{\text{Spec}} = 0 \]  
(33)

or

\[ Q_{ki} - Q_{ki}^{\text{Spec}} = 0 \]  
(34)

Which \( P_{ji}^{\text{Spec}} \) and \( Q_{ji}^{\text{Spec}} \) are the specified line active and line reactive power flow of first line and \( P_{ki}^{\text{Spec}} \) and \( Q_{ki}^{\text{Spec}} \) are the specified line active and line reactive power flow of second line.

2.5 GUPFC [6]

Figure (9) shows the equivalent circuit of the GUPFC. Similar to the active power exchange principle for the IPFC, real power can be exchanged among these shunt and series converters via the common DC link, and the sum of the real power exchange should be zero.
The equivalent circuit of a GUPFC consists of one controllable shunt injected voltage source and two controllable series injected voltage sources are shown in Fig. (10). In steady state operation, the main objective of a GUPFC is to control voltage and power flow.

According to the equivalent circuit of the GUPFC shown in Fig. (10), the power flow equations can be derived as:

\[
P_i = V_i^2 g_{ii} - V_i V_{sh} (g_{sh} \cos (\theta_i - \theta_{sh}) + b_{sh} \sin (\theta_i - \theta_{sh})) 
- \sum_n V_i V_n (g_{in} \cos (\theta_i - \theta_n) + b_{in} \sin (\theta_i - \theta_n)) 
- \sum_n V_i V_{serin} (g_{in} \cos (\theta_i - \theta_{serin}) + b_{in} \sin (\theta_i - \theta_{serin}))
\]  

(35)

\[
Q_i = - V_i^2 b_{ii} - V_i V_{sh} (g_{sh} \sin (\theta_i - \theta_{sh}) - b_{sh} \cos (\theta_i - \theta_{sh})) 
- \sum_n V_i V_n (g_{in} \sin (\theta_i - \theta_n) - b_{in} \cos (\theta_i - \theta_n)) 
- \sum_n V_i V_{serin} (g_{in} \sin (\theta_i - \theta_{serin}) - b_{in} \cos (\theta_i - \theta_{serin}))
\]  

(36)

\[
P_{ni} = V_i^2 g_{nn} - V_i V_n (g_{in} \cos (\theta_n - \theta_i) + b_{in} \sin (\theta_n - \theta_i)) 
+ V_j V_{serin} (g_{in} \cos (\theta_j - \theta_{serin}) + b_{in} \sin (\theta_j - \theta_{serin}))
\]  

(37)

\[
Q_{ni} = - V_i^2 b_{nn} - V_i V_j (g_{in} \sin (\theta_n - \theta_i) - b_{in} \cos (\theta_n - \theta_i)) 
+ V_n V_{serin} (g_{in} \sin (\theta_n - \theta_{serin}) - b_{in} \cos (\theta_n - \theta_{serin}))
\]  

(38)

Where

\[g_{sh} + j b_{sh} = 1 / Z_{sh}, g_{in} + j b_{in} = 1 / Z_{serin}, g_{nn} + j b_{nn} = 1 / Z_{serin}, g_{ii} = g_{sh} + \sum_n g_{in}, b_{ii} = b_{sh} + \sum_n b_{in}\]

The equivalent controllable injected voltage source hound constraints of the shunt converter are

\[
V_{shmin} \leq V_{sh} \leq V_{shmax}
\]

(39)

\[-\pi \leq \theta_{sh} \leq \pi
\]

(40)

Where \(V_{shmax}\), \(V_{shmin}\) are the maximal and minimal voltage limits of \(V_{sh}\).

The equivalent controllable injected voltage source magnitude and angle of the series converter are constrained by
\[ V_{ser_{in}}^{\text{min}} \leq V_{ser_{in}} \leq V_{ser_{in}}^{\text{max}} \quad (41) \]
\[ -\pi \leq \theta_{ser_{in}} \leq \pi \quad (42) \]

Where \( n = j, k \) \( V_{ser_{in}}^{\text{max}}, V_{ser_{in}}^{\text{min}} \) are the maximal and minimal voltage limits of \( V_{ser_{in}} \) respectively. The operating constraint representing the active power exchange among the converters via the common DC link is

\[ PE = P_{sh} - \Sigma_n P_{ser_{in}} \quad (n = j, k) \quad (43) \]

Where \( P_{sh} \) and \( P_{ser_{in}} \) is given by

\[ P_{sh} = \text{Re} (V_{sh} I_{sh}^*) \quad (44) \]
\[ P_{ser_{in}} = \text{Re} (V_{ser_{in}} I_{ser_{in}}^*) \quad (45) \]

Where \( I_{sh}^* \) and \( I_{ser_{in}}^* \) are conjugate of \( I_{sh} \) and \( I_{ser_{in}} \) respectively.

The GUPFC shown in Figs. (9) and (10) can control the voltage at bus i and the active and reactive power flows of the two lines i-j and i-k. Suppose the sending ends of the two lines are connected with the two buses j and k respectively. Then the active and reactive power flows of the two lines at the sending ends are — \( P_{ni}, Q_{ni} \) \( (n = j, k) \). The voltage and the active and reactive power flow control constraints of the GUPFC are:

\[ V_i - V_i^{\text{Spec}} = 0 \quad (46) \]
\[ P_{ni} - P_{ni}^{\text{Spec}} = 0 \quad (47) \]
\[ Q_{ni} - Q_{ni}^{\text{Spec}} = 0 \quad (48) \]

Where \( V_i^{\text{Spec}}, P_{ni}^{\text{Spec}} \) and \( Q_{ni}^{\text{Spec}} \) are specified bus voltage, active and reactive power flows, respectively.

### 3. THE COMBINED FACTS IN LOAD FLOW STUDIES

Incorporating the combined FACTS by a comprehensive Newton-Raphson power flow model into an existing MATLAB Newton-Raphson power flow algorithm is the subject of this section. This combined FACTS power flow model is modified to set control of the bus voltage magnitude and the active and reactive power flows of the transmission line. Based on this model it is possible to estimate the combined FACTS control variables and their ratings. The IEEE 30-bus system shown in Fig. (11) has been used as a test system to verify the use of the combined FACTS comprehensive NR power flow model. The system and line data are taken from [7].

#### 3.1 Studied Cases

Even cases of the combined FACTS power flow model will be studied in this section.

**Case 1:** This is a base case IEEE 30-bus system without FACTS.

**Case 2:** Two SSSCs in the System

Two SSSCs are set control the active power flow. The first SSSC is set control the active power flow of line 27-29 and the second SSSC is set control the active power flow of the line 24-23 this shown in Fig. (12).
Case 3: SSSC and IPFC in the System

The SSSC and the IPFC are set control the active and the reactive power flows of three transmission lines. The SSSC is set control the active power flow of line 27-29 and the IPFC is set control the active, the reactive power flows of the line 24-23 and the active power flow of line 24-25 this shown in Fig. (13).

Case 4: SSSC and GUPFC in the System

The SSSC and the GUPFC are set control the active and the reactive power flows of three transmission lines. The SSSC is set control the active power flow of line 27-29 and the GUPFC is set control the voltage magnitude of bus 24, the active and the reactive power flows of the lines 24-23 and 24-25 this shown in Fig (14).
Case 5: IPFC and GUPFC in the System

The IPFC and the GUPFC are set control the active and the reactive power flows of four transmission lines. The IPFC is set control the active and the reactive power flows of the line 10-20 and the active power flow of line 10-22. The GUPFC is set control the voltage magnitude of bus 24, the active and the reactive power flows of the lines 24-23 and 24-25 this shown in Fig. (15).

Case 6: STATCOM and UPFC in the System

The STATCOM is set control the voltage magnitude of bus 30. The UPFC is set control the voltage magnitude of bus 24, the active and the reactive power flows of line 24-23 this shown in Fig. (16).

Case 7: STATCOM and UPFC in the System

Similar to case 5 except that the UPFC is set control the voltage magnitude of bus 24, the active and the reactive power flows of line 24-25 this shown in Fig. (17).
Tables (1) and (2) show the base case IEEE 30-bus system without FACTS. Tables (3) and (4) show the bus voltage references of buses 30 and 24, the active and reactive power flow references of the lines 24-23, 24-25, 27-29, 10-20 and 10-22.

Table (1): The base case of the buses 30 and 24 and the lines 24-23, 24-25 and 27-29.

<table>
<thead>
<tr>
<th>$V_{30}$ (p.u)</th>
<th>$V_{24}$ (p.u)</th>
<th>$PL_{24-23}$ (MW)</th>
<th>$QL_{24-23}$ (MVAR)</th>
<th>$PL_{24-25}$ (MW)</th>
<th>$QL_{24-25}$ (MVAR)</th>
<th>$PL_{27-29}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.979</td>
<td>0.997</td>
<td>-2.04</td>
<td>-3.564</td>
<td>-1.571</td>
<td>-0.141</td>
<td>6.193</td>
</tr>
</tbody>
</table>

Table (2): The base case of the lines 10-20, 10-22 and the total system loss.

<table>
<thead>
<tr>
<th>$PL_{10-20}$ (MW)</th>
<th>$QL_{10-20}$ (MVAR)</th>
<th>$PL_{10-22}$ (MW)</th>
<th>$S_{Loss}$ (MVA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.599</td>
<td>1.872</td>
<td>7.377</td>
<td>39.326</td>
</tr>
</tbody>
</table>

Table (3): The bus voltage references of buses 30 and 24 and the active and reactive power flows references of lines 24-23, 24-25 and 27-29.

<table>
<thead>
<tr>
<th>$V_{ref}^{30}$ (p.u)</th>
<th>$V_{ref}^{24}$ (p.u)</th>
<th>$P_{ref}^{24-23}$ (MW)</th>
<th>$Q_{ref}^{24-23}$ (MVAR)</th>
<th>$P_{ref}^{24-25}$ (MW)</th>
<th>$Q_{ref}^{24-25}$ (MVAR)</th>
<th>$P_{ref}^{27-29}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>-1.8</td>
<td>-2</td>
<td>-0.8</td>
<td>-0.5</td>
<td>4</td>
</tr>
</tbody>
</table>

Table (4): The active and reactive power flows references of lines 10-20 and 10-22.

<table>
<thead>
<tr>
<th>$P_{ref}^{10-20}$ (MW)</th>
<th>$Q_{ref}^{10-20}$ (MVAR)</th>
<th>$P_{ref}^{10-22}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.5</td>
<td>5</td>
</tr>
</tbody>
</table>

Tables (5), (6) and (7) show the control variables of the combined FACTS, their ratings and the total system loss respectively.

From Table (7) the total system loss of case 6 is the least total system loss while the total system loss of case 3 is the most total system loss.
Table (5): the combined FACTS control variables of seven cases.

<table>
<thead>
<tr>
<th>No. of Case</th>
<th>Control Variables (p.u)</th>
<th>Control Variables (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$V_{ser\ (27-29)} = 0.0321$</td>
<td>$\theta_{ser\ (27-29)} = 48.975$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (24-23)} = 0.0008$</td>
<td>$\theta_{ser\ (24-23)} = 11.551$</td>
</tr>
<tr>
<td>3</td>
<td>$V_{ser\ (27-29)} = 0.0323$</td>
<td>$\theta_{ser\ (27-29)} = 48.122$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (24-23)} = 0.0114$</td>
<td>$\theta_{ser\ (24-23)} = 152.901$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (24-25)} = 0.0155$</td>
<td>$\theta_{ser\ (24-25)} = 290.41$</td>
</tr>
<tr>
<td>4</td>
<td>$V_{ser\ (27-29)} = 0.0322$</td>
<td>$\theta_{ser\ (27-29)} = 48.164$</td>
</tr>
<tr>
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<td>$V_{ser\ (24-23)} = 0.006$</td>
<td>$\theta_{ser\ (24-23)} = 167.31$</td>
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<td></td>
<td>$V_{ser\ (24-25)} = 0.0138$</td>
<td>$\theta_{ser\ (24-25)} = 278.087$</td>
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<tr>
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<td>$V_{sh\ (24)} = 1.0025$</td>
<td>$\theta_{sh\ (24)} = -16.6856$</td>
</tr>
<tr>
<td>5</td>
<td>$V_{ser\ (10-20)} = 0.0296$</td>
<td>$\theta_{ser\ (10-20)} = 263.574$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (10-22)} = 0.0079$</td>
<td>$\theta_{ser\ (10-22)} = 133.853$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (24-23)} = 0.0103$</td>
<td>$\theta_{ser\ (24-23)} = 224.716$</td>
</tr>
<tr>
<td></td>
<td>$V_{ser\ (24-25)} = 0.0177$</td>
<td>$\theta_{ser\ (24-25)} = 268.414$</td>
</tr>
<tr>
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<td>$V_{sh\ (24)} = 1.0020$</td>
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</tr>
<tr>
<td>6</td>
<td>$V_{ser\ (24-23)} = 0.0071$</td>
<td>$\theta_{ser\ (24-23)} = 129.735$</td>
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<tr>
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<td>$V_{sh\ (24)} = 1.0012$</td>
<td>$\theta_{sh\ (24)} = -16.391$</td>
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<td>$V_{sh\ (30)} = 1.003$</td>
<td>$\theta_{sh\ (30)} = 18.144$</td>
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<tr>
<td>7</td>
<td>$V_{ser\ (24-25)} = 0.0144$</td>
<td>$\theta_{ser\ (24-25)} = 220.867$</td>
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<td>$V_{sh\ (24)} = 1.0012$</td>
<td>$\theta_{sh\ (24)} = -16.587$</td>
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<td>$V_{sh\ (30)} = 1.0026$</td>
<td>$\theta_{sh\ (30)} = -17.804$</td>
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</tbody>
</table>

Table (6): the combined FACTS ratings of seven cases.

<table>
<thead>
<tr>
<th>No. of Case</th>
<th>Combined FACTS Ratings (MW)</th>
<th>Combined FACTS Ratings (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$P_{ser\ (27-29)} = 0.0000$</td>
<td>$Q_{ser\ (27-29)} = -0.1408$</td>
</tr>
<tr>
<td></td>
<td>$P_{ser\ (24-23)} = 0.0000$</td>
<td>$Q_{ser\ (24-23)} = 0.0030$</td>
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<tr>
<td>3</td>
<td>$P_{ser\ (27-29)} = 0.0000$</td>
<td>$Q_{ser\ (27-29)} = -0.1424$</td>
</tr>
<tr>
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<td>$Q_{ser\ (24-23)} = -0.0185$</td>
</tr>
<tr>
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<td>$P_{ser\ (24-25)} = 0.0241$</td>
<td>$Q_{ser\ (24-25)} = 0.0021$</td>
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<tr>
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<td>$Q_{ser\ (27-29)} = -0.1411$</td>
</tr>
<tr>
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<td>$P_{ser\ (24-23)} = -0.0098$</td>
<td>$Q_{ser\ (24-23)} = 0.00125$</td>
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<tr>
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<td>$P_{ser\ (24-25)} = 0.0108$</td>
<td>$Q_{ser\ (24-25)} = -0.0073$</td>
</tr>
<tr>
<td></td>
<td>$P_{sh\ (24)} = 0.001$</td>
<td>$Q_{sh\ (24)} = 2.4573$</td>
</tr>
</tbody>
</table>
INTEGRATING COMBINED FACTS IN LOAD ........

<table>
<thead>
<tr>
<th>No. of Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
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<tbody>
<tr>
<td>S_Loss</td>
<td>39.326</td>
<td>40.661</td>
<td>41.047</td>
<td>40.581</td>
<td>39.974</td>
<td>38.885</td>
<td>39.119</td>
</tr>
</tbody>
</table>

4. CONCLUSION

(a) Comprehensive mathematical modeling of STATCOM, SSSC, UPFC, IPFC and GUPFC has been presented in this paper. The models have been incorporated in a MATLAB power flow program based on NR algorithm.

(b) A comprehensive combined FACTS model has been presented in this paper. The model has been incorporated in a MATLAB power flow program based on NR algorithm.

(c) Seven cases of the combined FACTS power flow model have been presented in this paper by a comprehensive Newton-Raphson power flow model.

(d) This combined FACTS power flow model has been modified to set control the bus voltage magnitude, the active and reactive power flows of multi transmission lines. Based on this model we have been estimated the combined FACTS control variables and their ratings.

(e) The cases 6 and 7 make the total system loss less than the base case.

REFERENCES


إدخال نظام التيار المتناوب المرنة المتحدة في دراسات مسارات القدرة

في هذا البحث تم شرح نمذجة رياضية شاملة لـ IPFC و UPFC و SSSC و STATCOM و GUPFC و IPFC و UPFC و SSSC و STATCOM بطريقة خوارزمية نيوتن رافسون لسريان القدرة كما تم عمل أكثر من اتحاد بين هذه النظم وبعضها وإدخال هذا الاتحاد في برامج مسارات القدرة العادية لتوضيح طرق التحكم المختلفة التي تقوم بها هذه النظم المتحدة وحساب متغيرات التحكم وقدرة المحولات لهذه النظم المتحدة عند تشغيلها و تم تطبيق ذلك على نظام IEEE-30 bus test system المذكور وقد أظهرت النتائج القدرة العالية لهذه النظم المتحدة للتحكم في الجهود و القدرة الفعالة وغير الفعالة لعدة خطوط وكذلك المساهمة في تقليل فقد الكلي لنظام نقل القدرة.