Stability analysis for systems of nonlinear Hills equations

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Abstract:

Systems of nonlinear di erential equations with periodic coecients, which include Hills and Mathieus equations as examples in the linear limit, are important from a practical point of view. Nonlinear Hills equations model a variety of dynamical systems of interest to physics and engineering, in which perturbations enter as periodic modulations of their linear frequencies. As is well known, the stability properties of some fundamental periodic solutions of these systems is often an essential problem. The main purpose of this paper is to concentrate on one such class of nonlinear Hills equations and study the stability properties of some of their simplest periodic solutions analytically as well as numerically. To accomplish this task, we rst use an extension of the generalized averaging method to approximate these solutions and then apply the technique of multiple scaling to perform the stability analysis. A three-particle system with freefree boundary conditions is studied as an example. The accuracy of our results is tested, within the limits of rst-order perturbation theory, and is found to be well con rmed by numerical experiments. The stability analysis of these simple periodic solutions, though local in itself, can yield considerable information about more global properties of the dynamics, since it is in the vicinity of such solutions that the largest regions of regular or chaotic motion are usually observed, depending on whether the periodic solution is, respectively, stable or unstable.

Keywords:

Keywords: Hills equation; Periodic solution; Stable; Unstable; Nonlinear

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