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ON DECOMPOSITION OF CONTINUITY

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Abstract. The main purpose of this paper is to introduce the concepts of η-sets, $\eta\zeta$-sets, η-continuity and $\eta\zeta$-continuity and to obtain a decomposition of continuity.

1. Introduction

Tong [20] introduced the notions of A-sets and A-continuity in topological spaces and established a decomposition of continuity. In [21], he also introduced the notions of B-sets and B-continuity and used them to obtain a new decomposition of continuity. Recently, Ganster and Reilly [9] have improved Tong's decompostion result and provided a decomposition of A-continuity. A year later Ganster and Rielly [8] improved Tong's result [20] replacing A-continuity with LC-continuity and gave a decomposition of A-continuity. Quite recently, Przemski [18] has obtained some decompostions of continuity and weak continuity. Various decompositions of generalized types of continuous functions are given in [5, 6, 7, 10, 22].

In this paper, we introduce the notions of η-sets, $\eta\zeta$-sets, η-continuity and $\eta\zeta$-continuity and obtain another decomposition of continuity.

2. Preliminaries

Throughout this paper, spaces (X,τ) and (Y,σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed. For $A\subset X$, the closure, the interior, the boundary, the α-closure and the α-interior of A in X are denoted by $\overline{A}$, $\operatorname{Int} A$, $\operatorname{Fr} A$, $\operatorname{Cl}_\alpha(A)$ and $\operatorname{Int}_\alpha(A)$, respectively.

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The following definitions are all notions of generalized open sets used throughout this paper.

Definition 2.1. A subset A of a space X is called:

(1) a preopen set [15] if $A \subset \operatorname{Int} \overline{A}$ and a preclosed set if $\overline{\operatorname{Int} A} \subset A$,

(2) a semi-open set [12] if $A \subset \overline{\operatorname{Int} A}$ and a semi-closed set if $\operatorname{Int} \overline{A} \subset A$,

(3) an α-open set [17] if $A \subset \operatorname{Int} \overline{\operatorname{Int} A}$ and an α-closed set if $\overline{\operatorname{Int} \overline{A}} \subset A$,

(4) a β-open set [1] (=semi-preopen set [2]) if $A \subset \overline{\operatorname{Int} \overline{A}}$, and a β-closed set(=semi-preclosed set) if $\operatorname{Int} \overline{\operatorname{Int} A} \subset A$,

(5) a regular open set if $A = \operatorname{Int} \overline{A}$ and a regular closed set if $A = \overline{\operatorname{Int} A}$,

(6) an AB-set [5] $\big(= AB(X) \big)$ if $A = U \cap N$, where U is open and N is semi-regular, i.e. $\operatorname{Int} \overline{N} \subset N \subset \overline{\operatorname{Int} N}$,

(7) a C-set [10] if $A \in C(X) = \{U \cap N : U \in \tau, \operatorname{Int} \overline{\operatorname{Int} N} \subset N\}$,

(8) an A-set [20] if $A \in A(X) = \{U \cap N : U \in \tau, N = \overline{\operatorname{Int} N}\}$,

(9) a B-set [19] if $A \in B(X) = \{U \cap N : U \in \tau, \operatorname{Int} \overline{N} \subset N\}$,

(10) an LC-set [4] if $A \in LC(X) = \{U \cap N : U \in \tau, \overline{N} = N\}$,

(11) an A_7-set [22] if $A \in A_7(X) = \{U \cap N : U \in \tau, \overline{\operatorname{Int} N} \subset N\}$,

(12) a generalized α-closed [14] (written as gα-closed) in X if $\operatorname{Cl}_\alpha(A) \subset U$ whenever $A \subset U$ and U is open in X.

(13) nowhere dense [6] if $\operatorname{Int} \overline{A} = \emptyset$.

(14) an NDB-set [6] if A has nowhere dense boundary.

Definition 2.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be α-continuous [16] (resp. pre-continuous [15], semi-continuous [12], β-continuous [1], A_7-continuous [22], A-continuous [20], B-continuous [19], AB-continuous [5], LC-continuous [8]) if for each $V \in \sigma$, $f^{-1}(V)$ is an α-open set (resp. preopen set, semi-open set, β-open set, A_7-set, A-set, B-set, AB-set, LC-set). The function $f : (X, \tau) \to (Y, \sigma)$ is said to be α-irresolute [13] if for each α-open set V in Y, $f^{-1}(V)$ is an α-open set in X.

3. η-sets and $\eta\zeta$-sets

Definition 3.1. A subset A of a topological space (X, τ) is called an η-set (resp. an $\eta\zeta$-set) if $A = U \cap N$, where U is open and N is α-closed (resp. clopen) in (X, τ). The collection of all η-sets (resp. $\eta\zeta$-sets) in (X, τ) will be denoted by $\eta(X)$ (resp. $\eta\zeta(X)$).

The following implications are clear:

$$\begin{tikzcd}
\eta\zeta(X) \arrow[r] & A(X) \arrow[r] \arrow[dd] & LC(X) \arrow[d] & \\
 & & \eta(X) \arrow[r] \arrow[d] & A_7(X) \arrow[d] \\
 & AB(X) \arrow[r] & B(X) \arrow[r] & C(X)
\end{tikzcd}$$

Fig. 1

The converse of implications in Fig. 1 are not true as shown in the following examples.

EXAMPLE 3.1. Let $X=\{a,b,c\}$ and $\tau=\big\{\emptyset,\{a\},X\big\}$. Set $A=\{c\}$. It is easily observed that $A\in\eta(X)$ but $A\notin AB(X)$, $A\notin LC(X)$.

EXAMPLE 3.2. Let $X=\{a,b,c,d\}$ and $\tau=\big\{\emptyset,\{a\},\{b\},\{a,b\},X\big\}$. Set $A=\{b,c\}$. It is easily observed that $A\in AB(X)$ but $A\notin\eta(X)$. Also, set $B=\{b,c,d\}$. It is easily observed that $B\in A(X)$ but $B\notin\eta\zeta(X)$. Set $C=\{a,c\}$. One can deduce that $C\in AB(X)$ but $C\notin A_7(X)$.

EXAMPLE 3.3. Let $X=\{a,b,c,d\}$ and $\tau=\big\{\emptyset,\{a\},\{b,c\},\{a,b,c\},X\big\}$. Set $A=\{a,b\}$. It is easily observed that $A\in A_7(X)$ but $A\notin B(X)$.

REMARK 3.1. From the previous examples one can deduce that η-set and AB-set are independent.

THEOREM 3.1. *For a subset A of a topological space (X,τ), the following are equivalent:*

(1) *A is an η-set.*

(2) *$A=U\cap\mathrm{Cl}_\alpha(A)$ for some open set U.*

PROOF. (1) $\Rightarrow$ (2). Since A is an η-set, then $A=U\cap F$, where U is open and F is α-closed. So, $A\subset U$ and $A\subset F$. Hence, $\mathrm{Cl}_\alpha(A)\subset\mathrm{Cl}_\alpha(F)$. Therefore, $A\subset U\cap\mathrm{Cl}_\alpha(A)\subset U\cap\mathrm{Cl}_\alpha(F)=U\cap F=A$. Thus, $A=U\cap\mathrm{Cl}_\alpha(A)$.

(2) $\Rightarrow$ (1). It is obvious because $\mathrm{Cl}_\alpha(A)$ is α-closed.

THEOREM 3.2. *Let A be a subset of a topological space (X,τ). If $A\in\eta(X)$, then $\mathrm{Cl}_\alpha(A)-A$ is α-closed, $A\cup\big(X-\mathrm{Cl}_\alpha(A)\big)$ is α-open and $A\subseteqq\mathrm{Int}_\alpha\Big(A\cup\big(X-\mathrm{Cl}_\alpha(A)\big)\Big)$.*

PROOF. First, if $A\in\eta(X)$, then from Theorem 3.1 we have that $A=U\cap\mathrm{Cl}_\alpha(A)$ for some open set U. Therefore,

$$\mathrm{Cl}_\alpha(A)-A=\mathrm{Cl}_\alpha(A)-\big(U\cap\mathrm{Cl}_\alpha(A)\big)=\mathrm{Cl}_\alpha(A)\cap\Big(X-\big(U\cap\mathrm{Cl}_\alpha(A)\big)\Big)$$

$$= \mathrm{Cl}_\alpha(A) \cap \big((X-U) \cup \big(X - \mathrm{Cl}_\alpha(A)\big)\big)$$

$$= \big(\mathrm{Cl}_\alpha(A) \cap (X-U)\big) \cup \big(\mathrm{Cl}_\alpha(A) \cap \big(X - \mathrm{Cl}_\alpha(A)\big)\big)$$

$$= \big(\mathrm{Cl}_\alpha(A) \cap (X-U)\big) \cup \emptyset = \mathrm{Cl}_\alpha(A) \cap (X-U)$$

which is α-closed.

Second, since $\mathrm{Cl}_\alpha(A) - A$ is α-closed, then $X - (\mathrm{Cl}_\alpha(A) - A)$ is α-open. Therefore, $X - (\mathrm{Cl}_\alpha(A) - A) = X - \big(\mathrm{Cl}_\alpha(A) \cap (X-A)\big) = A \cup \big(X - \mathrm{Cl}_\alpha(A)\big)$.

Finally, since$A \cup \big(X - \mathrm{Cl}_\alpha(A)\big)$ is α-open, then $A \subseteqq A \cup \big(X - \mathrm{Cl}_\alpha(A)\big)$ $= \mathrm{Int}_\alpha\big(A \cup \big(X - \mathrm{Cl}_\alpha(A)\big)\big)$.

However, the converse of the above Theorem 3.2 need not be true as seen in the following example.

EXAMPLE 3.4. Let $X = \{a,b,c,d\}$ and $\tau = \big\{\emptyset, \{b,d\}, \{b,c,d\}, X\big\}$. Set $A = \{a,b,d\}$. It is easily observed that $\mathrm{Cl}_\alpha(A) - A = \{c\}$ is α-closed and $A \cup \big(X - \mathrm{Cl}_\alpha(A)\big) = A$ is α-open but $A \notin \eta(X)$.

THEOREM 3.3. *For a subset A of a topological space (X,τ), the following are equivalent:*

(1) *A is α-closed.*

(2) *A is an η-set and $g\alpha$-closed.*

PROOF. (1) $\Rightarrow$ (2). This is obvious.

(2) $\Rightarrow$ (1). Since A is an η-set, then $A = U \cap \mathrm{Cl}_\alpha(A)$, where U is an open set in X. So, $A \subset U$ and since A is $g\alpha$-closed, then $\mathrm{Cl}_\alpha(A) \subset U$. Therefore, $\mathrm{Cl}_\alpha(A) \subset U \cap \mathrm{Cl}_\alpha(A) = A$. Hence, A is α-closed.

THEOREM 3.4. *For a subset A of a topological space (X,τ) the following are equivalent:*

(1) *A is open.*

(2) *A is an $\eta\zeta$-set.*

(3) *A is α-open and an A-set* ([20, Theorem 3.2]).

(4) *A is pre-open and an A-set.*

(5) *A is α-open and an η-set.*

(6) *A is α-open and locally closed* ([9, Theorem 2]).

(7) *A is pre-open and locally closed* ([9, Theorem 2]).

(8) *A is pre-open and an η-set.*

(9) *A is pre-open and a B-set* ([21, Proposition 9]).

PROOF. (1) $\Rightarrow$ (2). Since A is open and $A = A \cap X$, where X is clopen, then A is an $\eta\zeta$-set.

(2) $\Rightarrow$ (3) and (3) $\Rightarrow$ (4) are trivial.

(4) $\Rightarrow$ (5). Since A is an A-set, then A is semi-open [6]. Now, A is both semi-open and pre-open and hence α-open [7]. The second part is trivial.

(5) $\Rightarrow$ (6). Since A is an η-set, then $A = U \cap \mathrm{Cl}_\alpha(A)$, where U is open. Therefore, since A is β-open, $\overline{\mathrm{Cl}_\alpha(A)} = \overline{(A \cup \overline{\mathrm{Int}\,\overline{A}})} = \overline{(\overline{\mathrm{Int}\,\overline{A}})} = \overline{Int\overline{A}}$ $= \mathrm{Cl}_\alpha(A)$. Hence, $\mathrm{Cl}_\alpha(A)$ is closed and A is locally closed.

(6) $\Rightarrow$ (7), (7) $\Rightarrow$ (8) and (8) $\Rightarrow$ (9) are trivial.

(9) $\Rightarrow$ (1). See Tong [21].

THEOREM 3.5. *For a subset A of a topological space (X, τ), the following are equivalent:*

(1) *A is an A-set.*

(2) *A is semi-open and an η-set.*

(3) *A is β-open and locally closed.*

(4) *A is β-open and an η-set.*

PROOF. (1) $\Rightarrow$ (2). This is trivial.

(2) $\Rightarrow$ (3). Since A is an η-set, then $A = U \cap \mathrm{Cl}_\alpha(A)$, where U is open. Also, since A is semi-open and so β-open, then $\overline{\mathrm{Cl}_\alpha(A)} = \overline{(A \cup \overline{\mathrm{Int}\,\overline{A}})}$ $= \overline{(\overline{\mathrm{Int}\,\overline{A}})} = \overline{\mathrm{Int}\,\overline{A}} = \mathrm{Cl}_\alpha(A)$. Hence, $\mathrm{Cl}_\alpha(A)$ is closed and A is locally closed.

(3) $\Rightarrow$ (4). This is trivial.

(4) $\Rightarrow$ (1). Since A is an η-set, then $A = U \cap \mathrm{Cl}_\alpha(A)$, where U is open. Also, since A is β-open, then $\mathrm{Cl}_\alpha(A) = \overline{\mathrm{Int}\,\overline{A}}$. So, $\overline{\mathrm{Int}\,\big(\,\mathrm{Cl}_\alpha(A)\big)} = \overline{\mathrm{Int}(\overline{\mathrm{Int}\,\overline{A}})}$ $= \overline{\mathrm{Int}\,\overline{A}} = \mathrm{Cl}_\alpha(A)$. Hence, $\mathrm{Cl}_\alpha(A)$ is regular closed and A is an A-set.

THEOREM 3.6. *For a subset A of a topological space (X, τ), the following are equivalent:*

(1) *A is α-closed.*

(2) *A is pre-closed and an η-set.*

(3) *A is pre-closed and a B-set.*

(4) *A is pre-closed and an NDB-set.*

PROOF. (1) $\Rightarrow$ (2). Every α-closed set is pre-closed. Since $A = A \cap X$, where A is α-closed and X is open, then A is an η-set.

(2) $\Rightarrow$ (3). Every η-set is a B-set.

(3) $\Rightarrow$ (4). Every B-set is an NDB-set [6].

(4) $\Rightarrow$ (1). Let A be an NDB-set and $B = X - A$. Then $\overline{\mathrm{Int}\,\mathrm{Fr}\,B} = \emptyset$ and $\mathrm{Int}\,\mathrm{Fr}\,B = \emptyset$. Therefore, $\mathrm{Int}\,\mathrm{Fr}\,B = \mathrm{Int}\,(\overline{B} \cap \overline{X - B}) = \mathrm{Int}\,\overline{B} \cap \mathrm{Int}\,\overline{X - B}$ $= \mathrm{Int}\,\overline{B} \cap (X - \overline{\mathrm{Int}\,B}) = \emptyset$. So, $\mathrm{Int}\,\overline{B} \subset \overline{\mathrm{Int}\,B}$. Since B is pre-open, then $B \subset \mathrm{Int}\,\overline{B} \subset \overline{\mathrm{Int}\,B}$. Therefore, B is semi-open and hence A is α-closed.

THEOREM 3.7. *For a space X, the following are equivalent:*

(1) *X is indiscrete.*

(2) *The η-sets in X are only the trivial ones.*

(3) *The $\eta\zeta$-sets in X are only the trivial ones.*

PROOF. (1) $\Rightarrow$ (2). If A is an η-set, then $A = U \cap B$, where U is open and B is α-closed. If $A \neq \emptyset$, then $U \neq \emptyset$ and by (1) $U = X$. Thus $A = B$ and so $A \supset \overline{\operatorname{Int} \overline{A}} = \overline{\operatorname{Int} X} = X$. Hence, $A = X$.

(2) $\Rightarrow$ (3). Every $\eta\zeta$-set is an η-set.

(3) $\Rightarrow$ (1). Since every open set is an $\eta\zeta$-set, then by (3) the open sets in X are only the trivial ones, i.e., X is indiscrete.

THEOREM 3.8. *For a space X, the following are equivalent:*

(1) *X is discrete.*

(2) *Every subset of X is an $\eta\zeta$-set.*

PROOF. (1) $\Rightarrow$ (2). By (1) any subset A of X is clopen. Hence, A is an $\eta\zeta$-set.

(2) $\Rightarrow$ (1). By (2) every singleton $\{x\}$ of X is an $\eta\zeta$-set and hence open. Thus X is discrete.

4. η-continuity and $\eta\zeta$-continuity

DEFINITION 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be η-continuous (resp. $\eta\zeta$-continuous) if the preimage of every open subset of Y is an η-set (resp. an $\eta\zeta$-set) in X.

The following diagram shows how η-continuity and $\eta\zeta$-continuity are related to some similar types of generalized continuity.

$$\begin{tikzcd}
\eta\zeta\text{-continuity} \arrow[r] & A\text{-continuity} \arrow[r] \arrow[dd] & LC\text{-continuity} \arrow[d] & \\
& & \eta\text{-continuity} \arrow[r] \arrow[d] & A_7\text{-continuity} \arrow[d] \\
& AB\text{-continuity} \arrow[r] & B\text{-continuity} \arrow[r] & C\text{-continuity}
\end{tikzcd}$$

Fig. 2

None of the implications in Fig. 2 is reversible as the following examples show.

EXAMPLE 4.1. Let $X = \{a, b, c, d\}$, $\tau = \big\{\emptyset, \{d\}, \{b, c\}, \{b, c, d\}, X\big\}$ and $\sigma = \big\{\emptyset, \{a, b\}, X\big\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is A_7-continuous but not B-continuous.

EXAMPLE 4.2. Let $X = \{a, b, c\}$, $\tau = \big\{\emptyset, \{a\}, \{b\}, \{a, b\}, X\big\}$ and $\sigma = \big\{\emptyset, \{b, c\}, X\big\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is A-continuous but not $\eta\zeta$-continuous.

Example 4.3. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b, c\}, X\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is LC-continuous but not A-continuous.

Example 4.4. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, X\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is AB-continuous but not A_7-continuous.

Example 4.5. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{c\}, X\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is η-continuous but neither LC-continuous nor AB-continuous.

Remark 4.1. From the previous examples we can see that η-continuity and AB-continuity are independent.

Theorem 4.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous and α-irresolute function. If B is an η-set in (Y, σ), then $f^{-1}(B)$ is an η-set in (X, τ).*

Proof. Let B be an η-set in (Y, σ). Then $B = U \cap F$, where U is open in (Y, σ) and F is α-closed in (Y, σ). Therefore, $f^{-1}(B) = f^{-1}(U \cap F) = f^{-1}(U) \cap f^{-1}(F)$. Since f is continuous, then $f^{-1}(U)$ is open in (X, τ). Also, since f is α-irresolute, then $f^{-1}(F)$ is α-closed in (X, τ). Hence, $f^{-1}(B)$ is an η-set in (X, τ).

Theorem 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*

(1) *f is continuous.*
(2) *f is $\eta\zeta$-continuous.*
(3) *f is α-continuous and A-continuous* ([20, Theorem 4.1]).
(4) *f is pre-continuous and A-continuous* ([9, Theorem 4]).
(5) *f is α-continuous and η-continuous.*
(6) *f is α-continuous and LC-continuous* ([9, Theorem 4]).
(7) *f is pre-continuous and LC-continuous* ([9, Theorem 4]).
(8) *f is pre-continuous and η-continuous.*
(9) *f is pre-continuous and B-continuous* ([21, Proposition 11]).

Proof. From Theorem 3.4, the proof is immediate.

Theorem 4.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*

(1) *f is A-continuous.*
(2) *f is semi-continuous and η-continuous.*
(3) *f is β-continuous and LC-continuous.*
(4) *f is β-continuous and η-continuous.*

Proof. From Theorem 3.5, the proof is immediate.

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