Supra b-irresolutness and Supra b-connectedness on Topological Space

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Abstract. In this paper, the concept of supra b-irresolute maps is introduced and several properties of it is investigated. Furthermore, the notion of supra b-connectedness is defined and researched by means of supra b-separated sets.

1. Introduction

Many authors introduced and studied various generalized properties and conditions containing some forms of sets in topological spaces. In 1983, Mashhour et al. [2] developed the supra topological spaces and studied s-continuous maps and $s^\star$-continuous maps. In 2008, Devi et al. [1] introduced and studied a class of sets and maps between topological spaces called supra α-open sets and supra α-continuous maps, respectively. In [3], the present authors introduced the concepts of supra b-open sets, supra b-continuous maps, supra b-open maps and supra b-closed maps and studied their properties. This paper is a continuation of the paper [3]. The purpose of this paper is to introduce the concept of supra b-irresolute maps. Besides developing the basic properties of these maps we study the notion of supra b-connectedness based on supra b-separated sets. We prove that supra b-connectedness is preserved by supra b-irresolute bijections.

Throughout this paper, (X, τ) , (Y, σ) and (Z, υ) (or simply, X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. All sets are assumed to be subsets of topological spaces. The closure

and the interior of a set A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subcollection $\mu \subset 2^X$ is called a supra topology [2] on a nonempty set X if $X \in \mu$ and μ is closed under arbitrary union. (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) and the complement of a supra open set is said to be supra closed. The supra closure of a set A, denoted by $Cl^\mu(A)$, is the intersection of supra closed sets including A. The supra interior of a set A,denoted by $Int^\mu(A)$, is the union of supra open sets included in A. The supra topology μ on X is said to be associated with the topology τ if $\tau \subset \mu$. A set A is said to be supra α-open [1] (resp. supra b-open [3]) if $A \subseteq Int^\mu(Cl^\mu(Int^\mu(A)))$ (resp. $A \subseteq Cl^\mu(Int^\mu(A)) \cup Int^\mu(Cl^\mu(A)))$. A set A is said to be supra b-closed if its complement $X - A$ is supra b-open. The supra $b-$closure of a set A, denoted by $Cl^\mu_b(A)$, is the intersection of supra $b-$closed sets including A. The supra $b-$interior of a set A,denoted by $Int^\mu_b(A)$, is the union of supra $b-$open sets included in A. A map $f : X \to Y$ is called a supra b-continuous map [3] if the inverse image of each open set in Y is a supra b-open set in X.

2. Supra b-irresolute Maps

In this section, we introduce a new type of maps called a supra b-irresolute maps and obtain some of their properties and characterizations.

Definition 2.1. A map $f : (X, \tau) \to (Y, \sigma)$ is called a supra b-irresolute map if the inverse image of each supra b-open set in Y is a supra b-open set in X.

Theorem 2.1. *Every supra b-irresolute map is a supra b-continuous map.*

Proof. Straightforward.   □

The converse of the above theorem is not true as shown in the following example.

Example 2.1. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$ be a topology on X. The supra topology μ is defined as follows: $\mu = \{X, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be a map defined as follows: $f(a) = b$, $f(b) = c$,and $f(c) = a$. The inverse image of the open set $\{a, b\}$ is $\{a, c\}$ which is a supra b-open set. Hence f is a supra b-continuous map. Furthermore, the inverse image of the supra b-open set $\{a\}$ is $\{c\}$ which is not a supra b-open set. Hence f is not a supra b-irresolute map.

Theorem 2.2. *Let (X, τ) and (Y, σ) be two topological spaces and μ and ν associated supra topologies with τ and σ, respectively. Let f be a map from X into Y. Then the following are equivalent:*
(1) f is a supra b-irresolute map;
(2) The inverse image of each supra b-closed set in Y is a supra b-closed set in X;
(3) $Cl^\mu_b(f^{-1}(B)) \subseteq f^{-1}(Cl^\nu_b(B))$ for every set B in Y;
(4) $f(Cl^\mu_b(A)) \subseteq Cl^\nu_b(f(A))$ for every set A in X;
(5) $f^{-1}(Int^\nu_b(B)) \subseteq Int^\mu_b(f^{-1}(B))$ for every B in Y.

Proof. (1)$\Rightarrow$(2): Let B be a supra b-closed set in Y. Then $Y - B$ is a supra b-open set in Y. Hence $f^{-1}(Y - B) = X - f^{-1}(B)$ is a supra b-open set in X. It follows that $f^{-1}(B)$ is a supra b-closed subset of X.

(2)$\Rightarrow$(3): Let B be any subset of Y. Since $Cl_b^\nu(B)$ is a supra b-closed set in Y, then $f^{-1}(Cl_b^\nu(B))$ is a supra b-closed set in X. Therefore $Cl_b^\mu(f^{-1}(B)) \subseteq Cl_b^\mu(f^{-1}(Cl_b^\nu(B))) = f^{-1}(Cl_b^\nu(B))$.

(3)$\Rightarrow$(4): Let A be any subset of X. By (3) we have $f^{-1}(Cl_b^\nu(f(A))) \supseteq Cl_b^\mu(f^{-1}(f(A))) \supseteq Cl_b^\mu(A)$.Therefore $f(Cl_b^\mu(A)) \subseteq Cl_b^\nu(f(A))$.

(4)$\Rightarrow$(5): Let B be any subset of Y. By (4), $f(Cl_b^\mu(X - f^{-1}(B))) \subseteq Cl_b^\nu(f(X - f^{-1}(B)))$ and $f(X - Int_b^\mu(f^{-1}(B))) \subseteq Cl_b^\nu(Y - B) = Y - Int_b^\nu(B)$. Therefore we have $X - Int_b^\mu(f^{-1}(B)) \subseteq f^{-1}(Y - Int_b^\nu(B))$ and hence $f^{-1}(Int_b^\nu(B)) \subseteq Int_b^\mu(f^{-1}(B))$.

(5)$\Rightarrow$(1): Let B be a supra b-open set in Y and $f^{-1}(Int_b^\nu(B)) \subseteq Int_b^\mu(f^{-1}(B))$. Then $f^{-1}(B) \subseteq Int_b^\mu(f^{-1}(B))$. But, $Int_b^\mu(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = Int_b^\mu(f^{-1}(B))$. Therefore $f^{-1}(B)$ is supra b-open in X.   □

Theorem 2.3. *If $f : X \to Y$ and $g : Y \to Z$ are supra b-irresolute maps, then $g \circ f : X \to Z$ is supra b-irresolute.*

Proof. Obvious.   □

3. Supra b-separated Sets

In this section, we shall research supra b-separated sets in supra topological spaces.

Definition 3.1. Let (X, μ) be a supra topological space and A, B be non-empty subsets of X. Then A and B are said to be supra b-separated if $A \cap Cl_b^\mu(B) = \phi$ and $Cl_b^\mu(A) \cap B = \phi$.

The following result is immediate from the above definition.

Theorem 3.1. *Let C and D are non-empty subsets of the supra b-separated sets A and B, respectively. Then C and D are also supra b-separated in X.*

Theorem 3.2. *Let A, B be non-empty subsets of X such that $A \cap B = \phi$ and A, B are either both supra b-open or both supra b-closed. Then A and B are supra b-separated.*

Proof. If both A and B are supra b-closed sets and $A \cap B = \phi$, then A and B are supra b- separated. Let now A and B be supra b-open and $A \cap B = \phi$. Then $A \subseteq X - B$. So $Cl_b^\mu(A) \subseteq Cl_b^\mu(X - B) = X - Int_b^\mu(B) = X - B$. Hence $Cl_b^\mu(A) \cap B = \phi$. Similarly, $A \cap Cl_b^\mu(B) = \phi$. Thus A and B are supra b-separated. □

Theorem 3.3. *Suppose that A and B are non-empty subsets of X such that either they are both supra b-open or they are both supra b-closed. If $C = A \cap (X - B)$ and $D = B \cap (X - A)$,then C and D are supra b-separated, provided they are non-empty.*

Proof. First suppose A and B are both supra b-open. Now $D = B \cap (X - A)$ implies $D \subseteq X - A$. Then $Cl_b^\mu(D) \subseteq Cl_b^\mu(X - A) = X - Int_b^\mu(A) = X - A$. Hence $A \cap Cl_b^\mu(D) = \phi$. Therefore $C \cap Cl_b^\mu(D) = \phi$. Similarly, $Cl_b^\mu(C) \cap D = \phi$. Thus C and D are supra b-separated.

Next, suppose that A and B are both supra b-closed sets. Then $C = A \cap (X - B)$ implies $C \subseteq A$. Hence $Cl_b^\mu(C) \subseteq Cl_b^\mu(A) = A$. Therefore $Cl_b^\mu(C) \cap D = \phi$. Similarly, $C \cap Cl_b^\mu(D) = \phi$. Thus C and D are supra b-separated.   □

Theorem 3.4. *Non-empty subsets A and B of X are supra b-separated if and only if there exist supra b-open sets U and V such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$, and $B \cap U = \phi$.*

Proof. Suppose that A and B are supra b-separated. Now, $A \cap Cl_b^\mu(B) = \phi$ and $Cl_b^\mu(A) \cap B = \phi$. Then $A \subseteq X - Cl_b^\mu(B) = U$ (say); and $B \subseteq X - Cl_b^\mu(A) = V$ (say). Since both $Cl_b^\mu(A)$ and $Cl_b^\mu(B)$ are supra b-closed, then both U and V are supra b-open. Therefore $A \subseteq Cl_b^\mu(A) = X - V$ and $B \subseteq Cl_b^\mu(B) = X - U$. Hence $A \cap V = \phi$ and $B \cap U = \phi$.

Conversely, let U and V be supra b-open sets such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$ and $B \cap U = \phi$. Then $X - U$ and $X - V$ are supra b-closed. Also, $A \cap V = \phi$ implies $A \subseteq X - V$. Therefore $Cl_b^\mu(A) \subseteq Cl_b^\mu(X - V) = X - V$. Hence $Cl_b^\mu(A) \cap V = \phi$ and $Cl_b^\mu(A) \cap B = \phi$. Similarly, $U \cap Cl_b^\mu(B) = \phi$ and $A \cap Cl_b^\mu(B) = \phi$. Thus A and B are supra b-separated.   □

4. Supra b-connectedness

In this section, we research supra b-connectedness by means of supra b-separated sets.

Definition 4.1. A subset A of X is said to be supra b-connected if it can't be represented as a union of two non-empty supra b-separated sets. If X is supra b-connected, then X is called a supra b-connected space.

Theorem 4.1. *A non-empty subset C of X is supra b-connected if and only if for every pair of supra b-separated sets A and B in X with $C \subseteq A \cup B$, one of the following possibilities holds:*
(a) $C \subseteq A$ and $C \cap B = \phi$,
(b) $C \subseteq B$ and $C \cap A = \phi$.

Proof. Let C be supra b-connected. Since $C \subseteq A \cup B$, then both $C \cap A = \phi$ and $C \cap B = \phi$ can not hold simultaneously. If $C \cap A \neq \phi$ and $C \cap B \neq \phi$, then by Theorem 3.1 they are also supra b-separated and $C = (C \cap A) \cup (C \cap B)$ which goes against the supra b-connectedness of C. Now, if $C \cap A = \phi$, then $C \subseteq B$, while $C \subseteq A$ holds if $C \cap B = \phi$.
Conversely, suppose that the given condition holds. Assume that C is not supra b-connected. Then there exist non-empty supra b-separated sets A and B in X

such that $C = A \cup B$. By hypothesis, either $C \cap A = \phi$ or $C \cap B = \phi$. So, either $A = \phi$ or $B = \phi$, none of which is true. Thus C is supra b-connected. □

Theorem 4.2. *The following properties are equivalent:*
(1) *A space X is not supra b-connected.*
(2) *There exist non-empty supra b-closed sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*
(3) *There exist non-empty supra b-open sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*

Proof. (1)⇒(2): Suppose that X is not supra b-connected. Then there exist non-empty subsets A and B such that $Cl_b^\mu(A) \cap B = A \cap Cl_b^\mu(B) = \phi$ and $A \cup B = X$. It follows that $Cl_b^\mu(A) = Cl_b^\mu(A) \cap (A \cup B) = (Cl_b^\mu(A) \cap A) \cup (Cl_b^\mu(A) \cap B) = A \cup \phi = A$. Hence A is a supra b-closed set. Similarly, B is supra b-closed. Thus (2) is held. (2)⇒(3) and (3)⇒(1): Obvious. □

Corollary 4.1. *The following are properties equivalent:*
(1) *A space X is supra b-connected.*
(2) *If A and B are supra b-open sets, $A \cup B = X$ and $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.*
(3) *If A and B are supra b-closed sets, $A \cup B = X$ and $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.*

Theorem 4.3. *For a subset G of X, the following conditions are equivalent:*
(1) *G is supra b-connected.*
(2) *There do not exist two supra b-closed sets A and B such that $A \cap G \neq \phi$, $B \cap G \neq \phi, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*
(3) *There do not exist two supra b-closed sets A and B such that $G \nsubseteq A$, $G \nsubseteq B, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*

Proof. (1)⇒(2): Suppose that G is supra b-connected and there exist two supra b-closed sets A and B such that $A \cap G \neq \phi$, $B \cap G \neq \phi, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $(A \cap G) \cup (B \cap G) = (A \cup B) \cap G = G$. Also, $Cl_b^\mu(A \cap G) \cap (B \cap G) \subseteq Cl_b^\mu(A) \cap (B \cap G) = A \cap B \cap G = \phi$. Similarly, $(A \cap G) \cap Cl_b^\mu(B \cap G) = \phi$. This shows that G is not supra b-connected, which is a contradiction.
(2)⇒(3): Suppose that there exist two supra b-closed sets A and B such that $G \nsubseteq A$, $G \nsubseteq B, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $A \cap G \neq \phi$ and $B \cap G \neq \phi$. This is a contradiction.
(3)⇒(1): Suppose that (3) is satisfied and G is not supra b-connected. Then there exist two non-empty supra b-separated sets C and D such that $G = C \cup D$. Thus $Cl_b^\mu(C) \cap D = C \cap Cl_b^\mu(D) = \phi$. Assume that $A = Cl_b^\mu(C)$ and $B = Cl_b^\mu(D)$. Hence $G \subseteq A \cup B$ and $Cl_b^\mu(C) \cap Cl_b^\mu(D) \cap (C \cup D) = (Cl_b^\mu(C) \cap Cl_b^\mu(D) \cap C) \cup (Cl_b^\mu(C) \cap Cl_b^\mu(D) \cap D) \subset (Cl_b^\mu(D) \cap C) \cup (Cl_b^\mu(C) \cap D) = \phi \cup \phi = \phi$. Now we prove that $G \nsubseteq A$ and $G \nsubseteq B$. In fact, if $G \subseteq A$, then $Cl_b^\mu(D) \cap G = B \cap G = B \cap (G \cap A) = \phi$, a contradiction. Thus $G \nsubseteq A$. Analogously we have $G \nsubseteq B$. This contradicts (3). Therefore G is supra b-connected. □

Corollary 4.2. *A space X is supra b-connected if and only if there do not exist two non-empty supra b-closed sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*

Theorem 4.4. *For a subset G of X, the following conditions are equivalent:*
(1) *G is supra b-connected.*
(2) *For any supra b-separated sets A and B with $G \subseteq A \cup B$, we have $G \cap A = \phi$ or $G \cap B = \phi$.*
(3) *For any supra b-separated sets A snd B with $G \subseteq A \cup B$, we have $G \subseteq A$ or $G \subseteq B$.*

Proof. $(1) \Rightarrow (2)$: Suppose that A and B are supra b-separated and $G \subseteq A \cup B$. Then by Theorem 3.1 we have $G \cap A$ and $G \cap B$ are also supra b-separated. Since G is supra b-connected and $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B)$, then $G \cap A = \phi$ or $G \cap B = \phi$.
$(2) \Rightarrow (3)$: If $G \cap A = \phi$, then $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B) = G \cap B$. So, $G \subseteq B$. Similarly, $G \cap B = \phi$ implies $G \subseteq A$.

$(3) \Rightarrow (1)$: Suppose that A and B are supra b-separated and $G = A \cup B$. Then by (3) either $G \subseteq A$ or $G \subseteq B$. If $G \subseteq A$, then $B = B \cap G \subseteq B \cap A \subseteq B \cap Cl_b^\mu(A) = \phi$. Similarly, if $G \subseteq B$, then $A = \phi$. So G can not be represented as a union of two non-empty supra b-separated sets. Therefore G is supra b-connected. $\square$

Theorem 4.5. *Let G be a supra b-connected subset of X. If $G \subseteq H \subseteq Cl_b^\mu(G)$, then H is also supra b-connected.*

Proof. Suppose that H is not supra b-connected. By Theorem 4.3 there exist two supra b-closed sets A and B such that $H \nsubseteq A, H \nsubseteq B, H \subseteq A \cup B$ and $A \cap B \cap H = \phi$. Since $G \subseteq H$, then $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Now we prove that $G \nsubseteq A$ and $G \nsubseteq B$. In fact, if $G \subseteq A$, then $Cl_b^\mu(G) \subseteq Cl_b^\mu(A) = A$. Therefore by hypothesis $H \subseteq A$ which is a contradiction. Hence $G \nsubseteq A$. Similarly, $G \nsubseteq B$. This contradicts that G is supra b-connected. $\square$

Theorem 4.6. *Let G and H be supra b-connected. If G and H are not supra b-separated, then $G \cup H$ is supra b-connected.*

Proof. Suppose that $G \cup H$ is not supra b-connected. By Theorem 4.3 there exist two supra b-closed sets A and B such that $G \cup H \nsubseteq A, G \cup H \nsubseteq B, G \cup H \subseteq A \cup B$ and $(G \cup H) \cap (A \cap B) = \phi$. So, either $G \nsubseteq A$ or $H \nsubseteq A$. Assume $G \nsubseteq A$. Then $G \subseteq B$ because G is supra b-connected. Hence $H \nsubseteq B$ and $H \subseteq A$. Thus $A \cap G \subseteq A \cap B \cap (G \cup H) = \phi$. Therefore $Cl_b^\mu(H) \cap G \subseteq Cl_b^\mu(A) \cap G = A \cap G = \phi$. Similarly, $H \cap Cl_b^\mu(G) = \phi$. This shows that G and H are supra b-separated, a contradiction. $\square$

Theorem 4.7. *Let $\{G_i\}_{i \in I}$ be a family of supra b-connected subsets of X. If there is $j \in I$ such that G_i and G_j are not supra b-separated for each $i \neq j$, then $\bigcup_{i \in I} G_i$ is supra b-connected.*

Proof. Suppose that $\bigcup_{i \in I} G_i$ is not supra b-connected. Then there exist non-empty

supra b-separated subsets A and B of X such that $\bigcup\limits_{i \in I} G_i = A \cup B$. For each $i \in I$, G_i is supra b-connected and $G_i \subseteq A \cup B$. Then by Theorem 4.1 either $G_i \subseteq A$ and $G_i \cap B = \phi$, or else $G_i \subseteq B$ and $G_i \cap A = \phi$. If possible, let for some $r, s \in I$ with $r \neq s, G_r \subseteq A$ and $G_s \subseteq B$. Then G_r, G_s being non-empty of supra b-separated sets which is not the case. Thus either $G_i \subseteq A$ with $G_i \cap B = \phi$ for each $i \in I$ or else $G_i \subseteq B$ with $G_i \cap A = \phi$ for each $i \in I$. In the first case $B = \phi$ (since $B \subseteq \bigcup\limits_{i \in I} G_i$) and in the second case $A = \phi$. Non of which is true. Thus $\bigcup\limits_{i \in I} G_i$ is supra b-connected. $\qquad\square$

Corollary 4.3. *Let $\{G_i\}_{i \in I}$ be a family of supra b-connected sets. If $\bigcap\limits_{i \in I} G_i \neq \phi$, then $\bigcup\limits_{i \in I} G_i$ is supra b-connected.*

Theorem 4.8. *A non-empty subset G of X is supra b-connected if and only if for any two elements x and y in G there exists a supra b-connected set H such that $x, y \in H \subseteq G$.*

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose by contrary that G is not supra b-connected. Then there exist two non-empty supra b-separated P, Q in X such that $G = P \cup Q$. Choose $x \in P$ and $y \in Q$. So, $x, y \in G$ and hence by hypothesis there exists a supra b-connected set H such that $x, y \in H \subseteq G$. Thus $H \cap P$ and $H \cap Q$ are non-empty supra b-separated sets with $H = (P \cap H) \cup (Q \cap H)$. This is contrary to the supra b-connectedness of H. $\qquad\square$

Theorem 4.9. *If $f : X \longrightarrow Y$ is a supra b-irresolute surjective map and C, D are supra b-separated sets in Y, then $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated in X.*

Proof. Since C and D are supra b-separated, we have $Cl_b^v(C) \cap D = C \cap Cl_b^v(D) = \phi$. By Theorem 2.2, $Cl_b^\mu(f^{-1}(C)) \cap f^{-1}(D) \subset f^{-1}(Cl_b^v(C)) \cap f^{-1}(D) = f^{-1}(Cl_b^v(C)) \cap D) = \phi$. Similarly, we have $f^{-1}(C) \cap Cl_b^\mu(f^{-1}(D)) = \phi$. Therefore, $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated. $\qquad\square$

Theorem 4.10. *If $f : X \longrightarrow Y$ is supra b-irresolute bijective and A is supra b-connected in X, then $f(A)$ is supra b-connected in Y.*

Proof. Suppose that $f(A)$ is not supra b-connected in Y. Then $f(A) = C \cup D$, where C and D are two non-empty supra b-separated in Y. By Theorem 4.9 $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated in X. Since f is bijective, then $A = f^{-1}(f(A)) = f^{-1}(C) \cup f^{-1}(D)$. Hence A is not supra b-connected in X. This is a contradiction. Thus $f(A)$ is supra b-connected in Y. $\qquad\square$

Theorem 4.11. *If $f : X \longrightarrow Y$ is a supra b-irresolute map and G is supra b-connected in X, then $f(G)$ is supra b-connected in Y.*

Proof. Suppose that $f(G)$ is not supra b-connected in Y. Then, by Theorem 4.3, there exist two supra b-closed sets A and B in Y such that $f(G) \not\subseteq A$, $f(G) \not\subseteq B$, $f(G) \subseteq A \cup B$ and $f(G) \cap A \cap B = \phi$. Hence we have that $G \not\subseteq f^{-1}(A)$, $G \not\subseteq f^{-1}(B)$, $G \subseteq f^{-1}(A) \cup f^{-1}(B)$ and $G \cap f^{-1}(A) \cap f^{-1}(B) \subseteq f^{-1}(f(G)) \cap f^{-1}(A) \cap f^{-1}(B) = f^{-1}(f(G) \cap A \cap B) = \phi$. This implies that G is not supra b-connected, a contradiction. Therefore $f(G)$ is supra b-connected in Y. $\qquad\square$

Corollary 4.4. *If $f : X \longrightarrow Y$ is a supra b-irresolute surjective map and X is supra b-connected, then so is Y.*

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