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A decomposition of soft continuity in soft topological spaces

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Abstract Introducing the notions of soft $\widetilde{t}$-sets, soft $\widetilde{t}^*$-sets, soft $\widetilde{\mathcal{B}}$-sets, soft $\widetilde{\alpha}^*$-sets and soft $\widetilde{C}$-sets in the setting of soft topological spaces, we study some of its properties and investigate the relationships between them besides considering some variants of continuous maps on soft topological spaces.

Keywords Soft set · Soft topology · Soft $\widetilde{\mathcal{B}}$-sets · Soft $\widetilde{C}$-sets · Soft weakly continuous

Mathematics Subject Classification 06D72 · 54A40

1 Introduction

In 1999, Molodtsov [22] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Later, he applied this theory to several directions [23–25]. The soft set theory has also been applied to many different fields (see, for example, [2,4,6,7,15,16,18–20,29]). Shabir and Naz [27] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Cağman et al. [8] defined the soft topology on a soft set and showed its related properties. Recently, authors studied some of basic concepts and properties of soft topological spaces, (see, for example, [5,8,10,11,

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13,21,26,27,30]). The notion of soft topology by Cağman et al. [8] is more general than by Shabir and Naz [27]. Therefore, we continue investigating the work of Cağman et al. [8] and follow their notations and mathematical formalism. In this paper, we introduce different types of soft subsets of soft topological space. It is organized as follows. The first section is the introduction. In Sect. 2 known basic notions and results concerning the theory of soft sets and soft topology are given. In Sect. 3 the notions of soft $\widetilde{\boldsymbol{t}}$-sets, soft $\widetilde{\boldsymbol{t}}^{*}$-sets and soft $\widetilde{\boldsymbol{\mathcal{B}}}$-sets are defined and some of their properties are studied. Also, some other characterizations of soft $\widetilde{\boldsymbol{t}}$-sets and soft $\widetilde{\boldsymbol{\mathcal{B}}}$-sets are given. In Sect. 4 the basic properties of soft $\widetilde{\boldsymbol{\alpha}}^{*}$-sets and soft $\widetilde{\boldsymbol{\mathcal{C}}}$-sets are introduced. Finally, in Sect. 5, the basic properties of soft weakly continuous, soft $\widetilde{\boldsymbol{\mathcal{B}}}$-continuous and soft $\widetilde{\boldsymbol{\mathcal{C}}}$-continuous are introduced and studied.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory. Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

Definition 2.1 [6,22] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \Phi$ if $x \notin A$. The set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2 [6] Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an A-universal soft set, denoted by $F_{\widetilde{A}}$. If $A = E$, then the A-universal soft set is called a universal soft set, denoted by $F_{\widetilde{E}}$.

Definition 2.3 [6] Let $F_A, F_B \in S(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.4 [6] Let $F_A, F_B \in S(U)$. Then, the soft union $F_A \widetilde{\cup} F_B$, the soft intersection $F_A \widetilde{\cap} F_B$, and the soft difference $F_A \widetilde{\setminus} F_B$, of F_A and F_B are defined by the approximate functions

$$f_{A\widetilde{\cup}B}(x) = f_A(x) \cup f_B(x), \quad f_{A\widetilde{\cap}B}(x) = f_A(x) \cap f_B(x), \quad f_{A\widetilde{\setminus}B}(x) = f_A(x) \setminus f_B(x),$$

respectively, and the soft complement $F_A^{\widetilde{c}}$ of F_A is defined by the approximate function $f_{A^{\widetilde{c}}}(x) = f_A^{c}(x)$, where $f_A^{\widetilde{c}}(x)$ is the complement of the set $f_A(x)$; that is, $f_A^{\widetilde{c}}(x) = U \widetilde{\setminus} f_A(x)$ for all $x \in E$. It is easy to see that $(f_A^{\widetilde{c}})^{\widetilde{c}} = F_A$ and $F_\Phi^{\widetilde{c}} = F_{\widetilde{E}}$.

Definition 2.5 [5] Let $F_A \in S(U)$. A soft set F_A is called a soft point if there exists a $x_0 \in U$ and $A \subseteq E$ such that $F_A(x) = \{x_0\}$, for all $x \in A$ and $F_A(x) = \emptyset$, for all $x \in E - A$. A soft point is denoted by $F_A^{x_0}$. A soft point $F_A^{x_0}$ is said to belong to a soft set G_B if $x_0 \in G_B(x)$, for each $x \in A$, and symbolically denoted by $F_A^{x_0} \in G_B$. The set of all soft points of U is denoted by $\mathbf{P}(U)$.

Definition 2.6 [16] Let $S(U, E)$ and $S(V, K)$ be the families of all soft sets over U and V, respectively. The mapping φ_ψ is called a soft mapping from U to V, denoted by $\varphi_\psi : S(U, E) \to S(V, K)$, where $\varphi : U \to V$ and $\psi : E \to K$ are two mappings.

(1) Let $F_A \in S(U)$, then the image of F_A under the soft mapping φ_ψ is the soft set over V denoted by $\varphi_\psi(F_A)$ and defined by

$$\varphi_\psi(F_A)(k) = \begin{cases} \bigcup_{x \in \psi^{-1}(k) \cap A} \varphi(F_A(x)), & \text{if } \psi^{-1}(k) \cap A \neq \emptyset; \\ \emptyset, & \text{otherwise .} \end{cases}$$

(2) Let $G_B \in S(V, K)$, then the pre-image of G_B under the soft mapping φ_ψ is the soft set over U denoted by $\varphi_\psi^{-1}(G_B)$, where

$$\varphi_\psi^{-1}(G_B)(x) = \begin{cases} \varphi^{-1}(G_B(\psi(x))), & \text{if } \psi(x) \in B; \\ \emptyset, & \text{otherwise .} \end{cases}$$

The soft mapping φ_ψ is called injective, if φ and ψ are injective. The soft mapping φ_ψ is called surjective, if φ and ψ are are surjective.

Proposition 2.7 [10] *Let φ_ψ be a soft mapping from $S(U, E)$ to $S(V, K)$, where $\varphi : U \to V$ and $\psi : E \to K$ are two mappings. Then for soft sets F_A, F_{A_1} over U and G_B, G_{B_1} over V we have:*

(1) *If $F_A \widetilde{\subseteq} F_{A_1}$, then $\varphi_\psi(F_A) \widetilde{\subseteq} \varphi_\psi(F_{A_1})$.*
(2) *If $G_B \widetilde{\subseteq} G_{B_1}$, then $\varphi_\psi^{-1}(G_B) \widetilde{\subseteq} \varphi_\psi^{-1}(G_{B_1})$.*
(3) $F_A \widetilde{\subseteq} \varphi_\psi^{-1}(\varphi_\psi(F_A))$.
(4) $\varphi_\psi(\varphi_\psi^{-1}(G_B)) \widetilde{\subseteq} G_B$.
(5) $\varphi_\psi^{-1}(G_B^{\widetilde{c}}) = (\varphi_\psi^{-1}(G_B))^{\widetilde{c}}$.

Definition 2.8 [5] Let $\varphi_\psi : (U, E) \to (V, K)$ and $\gamma_\delta : (V, K) \to (Z, L)$, then the composition of φ_ψ and γ_δ is denoted by $\gamma_\delta \circ \varphi_\psi$ and defined by $\gamma_\delta \circ \varphi_\psi := (\gamma_\delta \circ \varphi_\psi)_{(\delta \circ \psi)}$.

2.1 Soft topology

In this section, we give some basic results of soft topological spaces which we need next section.

Definition 2.9 [8] Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\widetilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

(1) $F_\Phi, F_{\widetilde{A}} \in \widetilde{\tau}$.
(2) $\{F_{A_i} \widetilde{\subseteq} F_A : i \in I \subseteq N\} \subseteq \widetilde{\tau} \Rightarrow \widetilde{\bigcup}_{i \in I} F_{A_i} \in \widetilde{\tau}$.
(3) $\{F_{A_i} \widetilde{\subseteq} F_A : 1 \leq i \leq n, n \in N\} \subseteq \widetilde{\tau} \Rightarrow \widetilde{\bigcap}_{i \in I}^{n} F_{A_i} \in \widetilde{\tau}$.

The pair $(F_A, \widetilde{\tau})$ is called a soft topological space. Every element of $\widetilde{\tau}$ is called a soft open set. Also, a soft set F_B is said to be soft closed if the soft set $F_B^{\widetilde{c}}$ is soft open.

Example 2.10 Let $\mathcal{R}$ be the real numbers, $A = \mathcal{R}^+$ be the positive real numbers and $(F_A)_\lambda = \{(x, (x - \lambda, x + \lambda)) : x \in A\}$. Consider the family $\widetilde{\tau} = \{(F_A)_\lambda : \lambda \in A\} \cup \{F_\Phi, F_{\widetilde{A}}\}$. Then, the pair $(\mathcal{R}, \widetilde{\tau})$ is a soft topological space.

Example 2.11 Let $\mathcal{R}$ be the real numbers and A be a countable set. Consider the family $\widetilde{\tau} = \{F_A : \bigcup_{e \in A} U \setminus F_A(e) \text{ is countable set}\} \cup \{F_\Phi\}$. Then, the pair $(\mathcal{R}, \widetilde{\tau})$ is a soft topological space.

Definition 2.12 [8] Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Then

(1) The soft interior of F_B, denoted F_B^o, is defined as the soft union of all soft open subsets of F_B.
(2) The soft closure of F_B, denoted $\overline{F_B}$, is defined as the soft intersection of all soft closed supersets of F_B.

Definition 2.13 Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Thus F_B is said to be

(1) A soft semi-open set [9] (resp. a soft semi-closed set [9]) if $F_B \widetilde{\subseteq} \overline{(F_B^o)}$ (resp. $(\overline{F_B})^o \widetilde{\subseteq} F_B$).
(2) A soft preopen set [12] (resp. a soft preclosed set [12]) if $F_B \widetilde{\subseteq} (\overline{F_B})^o$ (resp. $\overline{(F_B^o)} \widetilde{\subseteq} F_B$).
(3) A soft α-open set [1] (resp. a soft α-closed set [1]) if $F_B \widetilde{\subseteq} (\overline{(F_B^o)})^o$ (resp. $\overline{((\overline{F_B})^o)} \widetilde{\subseteq} F_B$).
(4) A soft regular open set [28] (resp. a soft regular closed set [28]) if $(\overline{F_B})^o = F_B$ (resp. $\overline{(F_B^o)} = F_B$).
(5) A soft β-closed set [3] if $(\overline{(F_B^o)})^o \widetilde{\subseteq} F_B$.

Definition 2.14 [10] Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces, $\varphi : U \to V$ and $\psi : E \to K$ be mappings, and $F_A^{x_0} \in \mathbf{P}(U)$. The mapping $\varphi_\psi : S(U, E) \to S(V, K)$ is soft continuous at $F_A^{x_0} \in \mathbf{P}(U)$ if for each $G_K \in N_{\widetilde{\tau}_2}(\varphi_\psi(F_A^{x_0}))$, there exists $H_E \in N_{\widetilde{\tau}_1}(F_A^{x_0})$ such that $\varphi_\psi(H_E) \widetilde{\subseteq} G_K$.

Definition 2.15 Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces. A soft mapping $\varphi_\psi : (U, \widetilde{\tau}_1, E) \to (V, \widetilde{\tau}_2, K)$ is said to be

(1) A soft continuous [5] if $\varphi_\psi^{-1}(G_B) \in \widetilde{\tau}_1, \forall\, G_B \in \widetilde{\tau}_2$.
(2) A soft semicontinuous [17] if $\varphi_\psi^{-1}(G_B)$ is soft semi-open set in $\widetilde{\tau}_1, \forall\, G_B \in \widetilde{\tau}_2$.
(3) A soft precontinuous [1] if $\varphi_\psi^{-1}(G_B)$ is soft preopen set in $\widetilde{\tau}_1, \forall\, G_B \in \widetilde{\tau}_2$.
(4) A soft α-continuous [1] if $\varphi_\psi^{-1}(G_B)$ is soft α-open set in $\widetilde{\tau}_1, \forall\, G_B \in \widetilde{\tau}_2$.

3 Soft $\widetilde{t}$-sets, soft $\widetilde{t^*}$-sets and soft $\widetilde{\mathcal{B}}$-sets

In this section we introduce the concepts of soft $\widetilde{t}$-sets, soft $\widetilde{t^*}$-sets and soft $\widetilde{\mathcal{B}}$-sets and investigate some of their basic properties.

Definition 3.1 Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Then, F_B is said to be a soft $\widetilde{t}$-set if $(\overline{F}_B)^o = F_B^o$.

Proposition 3.2 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Then*

(1) *A soft closed set is a soft $\widetilde{t}$-set.*
(2) *A soft regular open set is a soft $\widetilde{t}$-set.*

Proof The proof is straightforward. □

The converses of (1) and (2) in Proposition 3.2 are not true as illustrated by the following example.

Example 3.3 Let $U = \{u_1, u_2, u_3\}$, $A = \{x_1, x_2\}$ and $\widetilde{\tau} = \{F_\Phi, F_A, F_{A_1}\}$, where $F_{A_1} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Then $(F_A, \widetilde{\tau})$ is a soft topological space on F_A. Suppose that $G_A = \{(x_1, \{u_2, u_3\}), (x_2, \{u_3\})\}$. Then G_A is a soft $\widetilde{t}$-set but not a soft closed set. Also, $G_{A_1} = \{(x_1, \{u_2, u_3\}), (x_2, \{u_1, u_3\})\}$ is a soft $\widetilde{t}$-set but not a soft regular open set.

Generally speaking, a soft open set need not be a soft $\widetilde{t}$-set, as illustrated in the example below.

Example 3.4 Let $U = \{u_1, u_2\}$, $A = \{x_1, x_2\}$ and $\widetilde{\tau} = \{F_\Phi, F_A, F_{A_1}, F_{A_2}, F_{A_3}\}$, where $F_{A_1} = \{(x_1, U), (x_2, \{u_2\})\}$, $F_{A_2} = \{(x_1, \{u_1\}), (x_2, U)\}$, $F_{A_3} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Thus $(F_A, \widetilde{\tau})$ is a soft topological space on F_A, since F_{A_3} is a soft (open, semi-open, preopen, α-open) sets but not a soft $\widetilde{t}$-set. Also, note that $G_A = \{(x_1, \{u_2\}), (x_2, \phi)\}$ is a soft $\widetilde{t}$-set but not a soft (open, semi-open, preopen, α-open) sets.

Proposition 3.5 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B, F_C \widetilde{\subseteq} F_A$. If F_B and F_C is a soft $\widetilde{\boldsymbol{t}}$-sets, then $F_B \widetilde{\cap} F_C$ is a soft $\widetilde{\boldsymbol{t}}$-set.*

Proof The proof is straightforward. □

Proposition 3.6 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. If F_C is a soft $\widetilde{\boldsymbol{t}}$-set and $F_C \widetilde{\subseteq} F_B \widetilde{\subseteq} \overline{F_C}$, then F_B is a soft $\widetilde{\boldsymbol{t}}$-set.*

Proof The proof is straightforward. □

The following Propositions 3.7 and 3.8 below are straightforward and will be so stated without proof.

Proposition 3.7 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. F_B is a soft $\widetilde{\boldsymbol{t}}$-set if and only if F_B is a soft semi-closed set.*

Proposition 3.8 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. F_B is a soft regular open set if and only if F_B is a soft preopen set and a soft $\widetilde{\boldsymbol{t}}$-set.*

Definition 3.9 Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. F_B is said to be a soft $\widetilde{\boldsymbol{t}}^*$-set if $\overline{(F_B^o)} = \overline{F_B}$.

Theorem 3.10 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. The following assertions hold.*

(1) *A soft open set is a soft $\widetilde{\boldsymbol{t}}^*$-set.*
(2) *A soft regular closed set is a soft $\widetilde{\boldsymbol{t}}^*$-set.*
(3) *F_B is a soft $\widetilde{\boldsymbol{t}}$-set if and only if its complement is a soft $\widetilde{\boldsymbol{t}}^*$-set.*

Proof The proof is straightforward. □

The converses of (1) and (2) in Theorem 3.10 are not true as illustrated by the following example.

Example 3.11 Suppose that the soft topological space $(F_A, \widetilde{\tau})$ is the same as in Example 3.4. If $F_C = \{(x_1, \{u_1, u_3\}), (x_2, \{u_2, u_3\})\}$, then F_C is a soft $\widetilde{\boldsymbol{t}}^*$-set but not a soft open set. Note that F_{A_1} is a soft $\widetilde{\boldsymbol{t}}^*$-set but not a soft regular closed set.

The following Proposition 3.12 below is straightforward and will be so stated without proof.

Proposition 3.12 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B, F_C \widetilde{\subseteq} F_A$.*

(1) *If F_B and F_C is a soft $\widetilde{\boldsymbol{t}}^*$-sets, then $F_B \widetilde{\cup} F_C$ is a soft $\widetilde{\boldsymbol{t}}^*$-set.*
(2) *F_B is a soft $\widetilde{\boldsymbol{t}}^*$-set if and only if F_B is a soft semi-open set.*
(3) *F_B is a soft regular closed if and only if F_B is a soft preclosed set and a soft $\widetilde{\boldsymbol{t}}^*$-set.*

Definition 3.13 Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. F_B is said to be a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set if there is a soft open set F_C and a soft $\widetilde{\boldsymbol{t}}$-set F_D such that $F_B = F_C \widetilde{\cap} F_D$.

Proposition 3.14 *Let $(F_A, \widetilde{\tau})$ be a soft topological space on F_A. The following assertions hold.*

(1) *A soft open set is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set.*

(2) *A soft $\widetilde{\boldsymbol{t}}$-set is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set.*
(3) *A soft regular open set is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set.*
(4) *A soft closed set is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set.*

Proof (1) and (2) are straightforward.
(3) Follows from Proposition 3.2 (2) and Proposition 3.14 (2).
(4) Follows from Proposition 3.2 (1) and Proposition 3.14 (2). □

The converses of (1), (2), (3) and (4) in Proposition 3.14 are not true as illustrated by the following example.

Example 3.15 Suppose that the soft topological space $(F_A, \widetilde{\tau})$ is the same as in Example 3.3. Hence G_A is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set but not a soft (open, closed, regular open) set. Note also that F_{A_1} is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set but not a soft $\widetilde{\boldsymbol{t}}$-set.

Generally speaking, a soft α-open set need not be a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set as illustrated by the example below.

Example 3.16 The soft topological space $(F_A, \widetilde{\tau})$ is the same as in Example 3.3. Suppose that $F_B = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$. F_B is a soft (semi-open, preopen, α-open) sets but not a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set. Note that G_A is a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set but not a soft (semi-open, preopen, α-open) sets.

Proposition 3.17 *Let $(F_A, \widetilde{\tau})$ be a soft topological space on F_A. F_B is a soft open set if and only if F_B is a soft preopen set and a soft $\widetilde{\boldsymbol{\mathcal{B}}}$-set.*

Proof The proof is straightforward from Theorem 4.1 (1) [14] and Proposition 3.14 (1). □

4 Soft $\widetilde{\boldsymbol{\alpha}}^*$-sets and soft $\widetilde{\boldsymbol{C}}$-sets

In this section we introduce soft $\widetilde{\boldsymbol{\alpha}}^*$-sets and soft $\widetilde{\boldsymbol{C}}$-sets and investigate some of their basic properties.

Definition 4.1 Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. F_B is said to be a soft $\widetilde{\boldsymbol{\alpha}}^*$-set if $(\overline{(F_B^o)})^o = F_B^o$.

Proposition 4.2 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. The following statements are equivalent.*

(1) *F_B is a soft $\widetilde{\boldsymbol{\alpha}}^*$-set.*
(2) *F_B is a soft β-closed set.*
(3) *F_B^o is a soft regular-open set.*

Proof The proof is straightforward and is thus omitted. □

Proposition 4.3 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. The following assertions hold.*

(1) *If F_B is a soft $\widetilde{\boldsymbol{t}}$-set, then F_B is a soft $\widetilde{\boldsymbol{\alpha}}^*$-set.*
(2) *If F_B is a soft open set and a soft regular open set, then F_B is a soft $\widetilde{\boldsymbol{\alpha}}^*$-set.*
(3) *If F_B is a soft α-open set and a soft $\widetilde{\boldsymbol{\alpha}}^*$-set, then F_B is a soft regular open set.*
(4) *If F_B is a soft regular open set, then F_B is a soft α-open set.*

(5) *F_B is a soft semi-open set is a soft $\tilde{t}$-set if and only if F_B is a soft $\tilde{\alpha}^*$-set.*

Proof (1) and (2) are straightforward.
(3) Follows from Theorem 4.4 (1) [14].
(4) Follows from Remark 3.2 [28] and Theorem 4.1 (3) [14].
(5) Follows from Theorem 4.8 (1) [14].

Remark 4.4 The following example shows that

(1) The converse of Proposition 4.3 (1) is false,
(2) A soft open set need not be a soft $\tilde{\alpha}^*$-set, and
(3) The notion of a soft $\tilde{\alpha}^*$-set is different from that of a soft α-set.

Example 4.5 Let $U = \{u_1, u_2, u_3\}$, $A = \{x_1, x_2\}$ and $\tilde{\tau} = \{F_\Phi, F_A, F_{A_1}\}$, where $F_{A_1} = \{(x_1, \{u_1, u_3\}), (x_2, \{u_2, u_3\})\}$. Thus $(F_A, \tilde{\tau})$ is a soft topological space on F_A. Let $G_A = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$. Thus G_A is a soft $\tilde{\alpha}^*$-set but it is neither a soft $\tilde{t}$-set nor is a soft α-set. Note that F_{A_1} is a soft open set but not a soft $\tilde{\alpha}^*$-set. Every soft open set is a soft α-set but not a soft $\tilde{\alpha}^*$-set. Thus a soft $\tilde{\alpha}^*$-set is different from that of a soft α-set.

Remark 4.6 The union of two soft $\tilde{\alpha}^*$-sets need not be a soft $\tilde{\alpha}^*$-set. In Example 4.5, G_A and F_D are soft $\tilde{\alpha}^*$-sets, where $F_D = \{(x_1, \{u_3\}), (x_2, \{u_2, u_3\})\}$. However $G_A \tilde{\cup} F_D$ is not a soft $\tilde{\alpha}^*$-set.

Definition 4.7 Let $(F_A, \tilde{\tau})$ be a soft topological space on F_A. F_B is said to be a soft $\tilde{\mathcal{C}}$-set if there exist a soft open set F_C and a soft $\tilde{\alpha}^*$-set F_D such that $F_B = F_C \tilde{\cap} F_D$.

Proposition 4.8 *Let $(F_A, \tilde{\tau})$ be a soft topological space on F_A. The following assertions hold.*

(1) *A soft open set is a soft $\tilde{\mathcal{C}}$-set.*
(2) *A soft $\tilde{\alpha}^*$-set is a soft $\tilde{\mathcal{C}}$-set.*
(3) *A soft $\tilde{t}$-set is a soft $\tilde{\mathcal{C}}$-set.*
(4) *A soft $\tilde{\mathcal{B}}$-set is a soft $\tilde{\mathcal{C}}$-set.*
(5) *A soft regular open set is a soft $\tilde{\mathcal{C}}$-set.*

Proof (1) and (2) are straightforward.
(3) Follows from Proposition 4.3 (1) and Proposition 4.8 (2).
(4) Follows from Proposition 4.3 (1).
(5) Follows from Proposition 3.14 (3) and Proposition 4.8 (4). □

The converses of (1), (2), (3), (4) and (5) in Proposition 4.8 are not true as illustrated by the following example.

Example 4.9 Suppose that the soft topological space $(F_A, \tilde{\tau})$ is the same as in Example 4.5. It can be seen that G_A is a soft $\tilde{\mathcal{C}}$-set but not a soft open set, a soft $\tilde{t}$-set, a soft $\tilde{\mathcal{B}}$-set, nor a soft regular open set. Note that F_{A_1} is a soft $\tilde{\mathcal{C}}$-set but not a soft $\tilde{\alpha}^*$-set.

Proposition 4.10 *Let $(F_A, \tilde{\tau})$ be a soft topological space on F_A. F_B is a soft open set if and only if F_B is a soft α-open set and a soft $\tilde{\mathcal{C}}$-set.*

Proof The proof is straightforward from Theorem 4.1 (3) [14], Proposition 4.8 (1) and Theorem 4.10 [14]. □

Remark 4.11 The notion of a soft α-open set is different from that of a soft $\widetilde{\mathcal{C}}$-set. In Example 4.5, G_A is a soft $\widetilde{\mathcal{C}}$-set but not a soft α-open set. The following example shows that a soft α-open set need not be a soft $\widetilde{\mathcal{C}}$-set.

Example 4.12 Let $U = \{u_1, u_2, u_3\}$, $A = \{x_1, x_2\}$ and $\widetilde{\tau} = \{F_\Phi, F_A, F_{A_1}\}$, where $F_{A_1} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Thus $(F_A, \widetilde{\tau})$ is a soft topological space on F_A. Let $G_A = \{(x_1, \{u_1, u_3\}), (x_2, \{u_2, u_3\})\}$. Thus G_A is a soft α-open set but not a soft $\widetilde{\mathcal{C}}$-set.

5 Decomposition of soft continuity

In this section, we introduce the notion of soft weakly continuous, soft $\widetilde{\mathcal{B}}$-continuous and soft $\widetilde{\mathcal{C}}$-continuous of functions induced by two mappings $\phi : U \to V$ and $\psi : E \to K$ on soft topological spaces $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$.

Definition 5.1 Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces, $\varphi : U \to V$ and $\psi : E \to K$ be mappings, and $F_A^{x_0} \in \mathbf{P}(U)$.

(1) The map $\varphi_\psi : S(U, E) \to S(V, K)$ is soft weakly continuous at $F_A^{x_0} \in \mathbf{P}(U)$ if for each $G_K \in N_{\widetilde{\tau}_2}(\varphi_\psi(F_A^{x_0}))$, there exists $H_E \in N_{\widetilde{\tau}_1}(F_A^{x_0})$ such that $\varphi_\psi(H_E)\widetilde{\subseteq}\overline{G_K}$.
(2) The map $\varphi_\psi : S(U, E) \to S(V, K)$ is soft weakly continuous on U if φ_ψ is soft weakly continuous at each soft point in U.

Proposition 5.2 *Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces. The following statements are equivalent.*

(1) *The map φ_ψ is soft weakly continuous;*
(2) *For each $G_K \in \widetilde{\tau}_2$, $\overline{(\varphi_\psi^{-1}(G_K))}\widetilde{\subseteq}\varphi_\psi^{-1}(\overline{G_K})$;*
(3) *For each $G_K \in \widetilde{\tau}_2$, $\varphi_\psi^{-1}(G_K)\widetilde{\subseteq}(\varphi_\psi^{-1}(\overline{G_K}))^o$.*

Proof (1)⇒(2) Suppose there exists a point $F_A^{x_0} \in \overline{(\varphi_\psi^{-1}(G_K))} \setminus \varphi_\psi^{-1}(\overline{G_K})$. Thus $\varphi_\psi^{-1}(F_A^{x_0}) \notin \overline{G_K}$. Hence there exists $M_K \in N_{\widetilde{\tau}_2}(\varphi_\psi(F_A^{x_0}))$ such that $M_K \widetilde{\cap} G_K = F_\Phi$. Since $G_K \in \widetilde{\tau}_2$, we have $G_K \widetilde{\cap} \overline{M_K} = F_\Phi$. Since φ_ψ is soft weakly continuous, there exists $H_E \in N_{\widetilde{\tau}}(F_A^{x_0})$ such that $\varphi_\psi(H_E)\widetilde{\subseteq}\overline{M_K}$. Thus we obtain $\varphi_\psi(H_E)\widetilde{\cap}G_K = F_\Phi$. On the other hand, since $F_A^{x_0} \in \overline{(\varphi_\psi^{-1}(G_K))}$, we have $H_E\widetilde{\cap}\varphi_\psi^{-1}(G_K) \neq F_\Phi$ and hence $\varphi_\psi(H_E)\widetilde{\cap}(G_K) \neq F_\Phi$. We have a contradiction. Thus we have $\overline{(\varphi_\psi^{-1}(G_K))}\widetilde{\subseteq}\varphi_\psi^{-1}(\overline{G_K})$.
(2)⇒(3) Let $G_K \in \widetilde{\tau}_2$ and let $M_K = V \backslash \overline{G_K}$. $\overline{(\varphi_\psi^{-1}(M_K))}\widetilde{\subseteq}\varphi_\psi^{-1}(\overline{M_K})$ implies $((\varphi_\psi^{-1}(\overline{M_K}))^o)^{\widetilde{c}}\widetilde{\subseteq}(\varphi_\psi^{-1}((\overline{G_K})^o)))^{\widetilde{c}}\widetilde{\subseteq}(\varphi_\psi^{-1}(G_K)))^{\widetilde{c}}$. Thus $\varphi_\psi^{-1}(G_K)\widetilde{\subseteq}(\varphi_\psi^{-1}(\overline{G_K}))^o$.
(3)⇒(1) Let $F_A^{x_0} \in \mathbf{P}(U)$ and $\varphi_\psi(F_A^{x_0}) \in G_K$. Thus $F_A^{x_0} \in \varphi_\psi^{-1}(G_K)\widetilde{\subseteq}(\varphi_\psi^{-1}(\overline{G_K}))^o$. Let $H_E = (\varphi_\psi^{-1}(\overline{G_K}))^o$. $\varphi_\psi(H_E) = \varphi_\psi((\varphi_\psi^{-1}(\overline{G_K}))^o)\widetilde{\subseteq}\varphi_\psi(\varphi_\psi^{-1}(\overline{G_K}))\widetilde{\subseteq}\overline{G_K}$. Hence φ_ψ is soft weakly continuous.

□

Proposition 5.3 *Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces, $\varphi : U \to V$ and $\psi : E \to K$ be a mapping. If φ_ψ is soft continuous, then φ_ψ is soft weakly continuous.*

Proof The proof is obvious and is thus omitted. □

The converse of Proposition 5.3 are not true as illustrated by the following example.

Example 5.4 Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, E = \{x_1, x_2\}$, and $K = \{y_1, y_2\}$. We consider the soft topology $\widetilde{\tau}_1 = \{H_\Phi, H_U, \{(x_1, \{u_3\}), (x_2, \{u_1, u_2\})\}, \{(x_1, \phi), (x_2, \{u_3\})\}, \{(x_1, \{u_3\}), (x_2, U)\}\}$ over U and the soft topology $\widetilde{\tau}_2 = \{G_\Phi, G_V, \{(y_1, \{v_1\}), (y_2, \{v_2\})\}\}$ over V. Let $\varphi : U \to V$ be the map such that $\varphi(u_1) = v_2$ and $\varphi(u_2) = \varphi(u_3) = v_1$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. Thus $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft weakly continuous but not a soft continuous.

Proposition 5.5 *Let $\varphi_\psi : (U, \widetilde{\tau}_1, E) \to (V, \widetilde{\tau}_2, K)$ and $\gamma_\delta : (V, \widetilde{\tau}_2, K) \to (Z, \widetilde{\tau}_3, L)$ are soft weakly continuous and either φ_ψ or γ_δ is soft continuous. Therefore $\gamma_\delta \circ \varphi_\psi$ is soft weakly continuous.*

Proof Suppose first that φ_ψ is a soft continuous and γ_δ is a soft weakly continuous. For a soft set F_B on Z, $(\gamma_\delta \circ \varphi_\psi)^{-1}(F_B)(x) = ((\gamma_\delta \circ \varphi_\psi)_{(\delta\circ\psi)})^{-1}(F_B)(x) = (\gamma \circ \varphi)^{-1}(F_C(\delta(\psi(x)))) = (\varphi^{-1}\,(\gamma^{-1}(F_B(\delta(\psi(x))))) = \varphi_\psi^{-1}\,(\gamma_\delta^{-1}(F_B))(x) \widetilde{\subseteq} \varphi_\psi^{-1}\,(\gamma_\delta^{-1}(\overline{F_B}))^o(x) \widetilde{\subseteq} (\varphi_\psi^{-1}\,(\gamma_\delta^{-1}(\overline{F_B})))^o(x) = ((\gamma \circ \varphi)^{-1}(\overline{F_B}))^o(x)$. Hence, we have $(\gamma_\delta \circ \varphi_\psi)^{-1}(F_B) \widetilde{\subseteq} ((\gamma \circ \varphi)^{-1}(\overline{F_B}))^o$. Therefore $\gamma_\delta \circ \varphi_\psi$ is a soft weakly continuous. Now suppose that φ_ψ is a soft weakly continuous and γ_δ is a soft continuous. For a soft set F_B on Z, $(\gamma_\delta \circ \varphi_\psi)^{-1}(F_B)(x) = \varphi_\psi^{-1}\,(\gamma_\delta^{-1}(F_B))(x) \widetilde{\subseteq} (\varphi_\psi^{-1}(\overline{\gamma_\delta^{-1}(F_B)}))^o(x) \widetilde{\subseteq} (\varphi_\psi^{-1}(\gamma_\delta^{-1}(\overline{F_B})))^o(x) \widetilde{\subseteq} (\gamma_\delta \circ \varphi_\psi)^{-1}(\overline{F_B})(x)$. Hence, we have $(\gamma_\delta \circ \varphi_\psi)^{-1}(F_B) \widetilde{\subseteq} ((\gamma_\delta \circ \varphi_\psi)^{-1}(\overline{F_B}))^o$. Therefore $\gamma_\delta \circ \varphi_\psi$ is a soft weakly continuous. □

Definition 5.6 Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces.

(1) A soft mapping $\varphi_\psi : (U, \widetilde{\tau}_1, E) \to (V, \widetilde{\tau}_2, K)$ is said to be a soft $\widetilde{\mathcal{B}}$-continuous if $\varphi_\psi^{-1}(G_B)$ is a soft $\widetilde{\mathcal{B}}$-set in $\widetilde{\tau}_1$, $\forall\, G_B \in \widetilde{\tau}_2$.
(2) A soft mapping $\varphi_\psi : (U, \widetilde{\tau}_1, E) \to (V, \widetilde{\tau}_2, K)$ is said to be a soft $\widetilde{\mathcal{C}}$-continuous if $\varphi_\psi^{-1}(G_B)$ is a soft $\widetilde{\mathcal{C}}$-set in $\widetilde{\tau}_1$, $\forall\, G_B \in \widetilde{\tau}_2$.

Proposition 5.7 *Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces, $\varphi : U \to V$ and $\psi : E \to K$ be a mapping. The following assertions hold.*

(1) *If φ_ψ is a soft continuous, then φ_ψ is a soft $\widetilde{\mathcal{B}}$-continuous.*
(2) *If φ_ψ is a soft continuous, then φ_ψ is a soft $\widetilde{\mathcal{C}}$-continuous.*
(3) *If φ_ψ is a soft $\widetilde{\mathcal{B}}$-continuous, then φ_ψ is a soft $\widetilde{\mathcal{C}}$-continuous.*

Proof (1) Follows from Proposition 3.14 (1).
(2) Follows from Proposition 4.8 (1).
(3) Follows from Proposition 4.8 (4).

□

The converses of (1), (2) and (3) in Proposition 5.7 are not true as illustrated by the following examples.

Example 5.8 Suppose we have the same sets as in Example 5.4. Define the map $\varphi : U \to V$ to be $\varphi(u_1) = \varphi(u_2) = v_1$ and $\varphi(u_3) = v_2$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_1$ and $\psi(x_2) = y_2$. Thus $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft $\widetilde{\mathcal{B}}$-continuous and a soft $\widetilde{\mathcal{C}}$-continuous but not a soft continuous.

Example 5.9 Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, E = \{x_1, x_2\}$, and $K = \{y_1, y_2\}$. We consider the soft topology $\widetilde{\tau}_1 = \{H_\Phi, H_U, \{(x_1, \{u_1\}), (x_2, \{u_2\})\}\}$ over U and the soft topology $\widetilde{\tau}_2 = \{G_\Phi, G_V, \{(y_1, \{v_1\}), (y_2, \{v_2\})\}\}$ over V. Let $\varphi : U \to V$ be the map such that $\varphi(u_1) = \varphi(u_2) = v_1$ and $\varphi(u_3) = v_2$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_1$ and $\psi(x_2) = y_2$. Hence $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is $\widetilde{\mathcal{C}}$-continuous but not a soft $\widetilde{\mathcal{B}}$-continuous.

Proposition 5.10 *Let $(U, \widetilde{\tau}_1, E)$ and $(V, \widetilde{\tau}_2, K)$ be two soft topological spaces, $\varphi : U \to V$ and $\psi : E \to K$ be a mapping. The following assertions hold.*

(1) *A soft mapping φ_ψ is a soft continuous if and only if it is both a soft precontinuous and a soft $\widetilde{\mathcal{B}}$-continuous.*
(2) *A soft mapping φ_ψ is a soft continuous if and only if it is both a soft α-continuous and a soft $\widetilde{\mathcal{C}}$-continuous.*

Proof (1) Follows from Proposition 3.17.
(2) Follows from Proposition 4.10.

□

Remark 5.11 A soft $\widetilde{\mathcal{B}}$-continuity is independent of soft semicontinuity, soft precontinuity and soft α-continuity as shown in the examples below.

Example 5.12 In Example 5.8, $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft $\widetilde{\mathcal{B}}$-continuous but not a soft precontinuous and soft α-continuous.

Example 5.13 Let $U = \{u_1, u_2\}$, $V = \{v_1, v_2\}$, $E = \{x_1, x_2\}$, and $K = \{y_1, y_2\}$. We consider the soft topology $\widetilde{\tau}_1 = \{H_\Phi, H_U, \{(x_1, U), (x_2, \{u_2\})\}, \{(x_1, \{u_1\}), (x_2, U)\}, \{(x_1, \{u_1\}), (x_2, \{u_2\})\}\}$ over U and the soft topology $\widetilde{\tau}_2 = \{G_\Phi, G_V, \{(y_1, \{v_1\}), (y_2, \{v_2\})\}\}$ over V. Let $\varphi : U \to V$ be the map such that $\varphi(u_1) = v_2$ and $\varphi(u_2) = v_1$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. It can be seen that $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft $\widetilde{\mathcal{B}}$-continuous but not a soft semicontinuous.

Example 5.14 In Example 5.9, $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft precontinuous but not a soft $\widetilde{\mathcal{B}}$-continuous.

Example 5.15 Suppose we have the same sets as in Example 5.9. Define the map $\varphi : U \to V$ as $\varphi(u_2) = v_1$ and $\varphi(u_1) = \varphi(u_3) = v_2$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. It can be seen that $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft semicontinuous and a soft α-continuous but not a soft $\widetilde{\mathcal{B}}$-continuous.

Remark 5.16 A soft $\widetilde{\mathcal{B}}$-continuity is independent of soft weakly continuity as illustrated in the examples below.

Example 5.17 In Example 5.4, $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft weakly continuous but not a soft $\widetilde{\mathcal{B}}$-continuous.

Example 5.18 Let $U = \{u_1, u_2\}$, $V = \{v_1, v_2\}$, $E = \{x_1, x_2\}$, and $K = \{y_1, y_2\}$. We consider the soft topology $\widetilde{\tau}_1 = \{H_\Phi, H_U, \{(x_1, U), (x_2, \{u_2\})\}, \{(x_1, \{u_1\}), (x_2, U)\}, \{(x_1, \{u_1\}), (x_2, \{u_2\})\}\}$ over U and the soft topology $\widetilde{\tau}_2 = \{G_\Phi, G_V, \{(y_1, \{v_1\}), (y_2, \{v_2\}), \{(y_1, \{v_2\}), (y_2, \{v_1\})\}\}$ over V. Let $\varphi : U \to V$ be the map such that $\varphi(u_1) = v_2$ and $\varphi(u_2) = v_1$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. Thus $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft $\widetilde{\mathcal{B}}$-continuous but not a soft weakly continuous.

Remark 5.19 A soft $\widetilde{\mathcal{C}}$-continuity is independent of soft weakly continuity as illustrated in the examples below.

Example 5.20 In Example 5.18, $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft $\widetilde{\mathcal{C}}$-continuous but not a soft weakly continuous.

Example 5.21 Suppose we have the same sets as in Example 5.9. Define the map $\varphi : U \to V$ as $\varphi(u_1) = \varphi(u_3) = v_1$ and $\varphi(u_2) = v_2$ and $\psi : E \to K$ be the map such that $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. Thus $\{(y_1, \{v_1\}), (y_2, \{v_2\})\}$ is a soft weakly continuous but not a soft $\widetilde{\mathcal{C}}$-continuous.

6 Conclusion

In this paper, we have introduced the concepts of $\widetilde{t}$-sets, soft $\widetilde{t}^*$-sets, soft $\widetilde{\mathcal{B}}$-sets, soft $\widetilde{\alpha}^*$-sets and soft $\widetilde{C}$-sets in topological spaces and some of their properties are studied. We also defined soft weakly continuous, soft $\widetilde{\mathcal{B}}$-continuous and soft $\widetilde{\mathcal{C}}$-continuous and have established several interesting properties. In the end, we have developed a new structure and will carry out more theoretical research to promote a general framework for practical applications.

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