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L-FUZZIFYING SOFT PREPROXIMITIES

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Abstract: The subject of the paper is the description of the category as objects L-fuzzifying soft preproximity spaces with structure preserving morphisms. We investigate the functorial relations between L-fuzzifying soft preproximity spaces and L-fuzzifying soft topological spaces.

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1. Introduction

The real world is too complex for our immediate and direct understanding. We create "models" of reality that are simplifications of aspects of the real word. In 1999 D. Molodtsov [14] introduced the concept of a soft set and started to develop basic of the theory as a new approach for modeling uncertainties. Research works on soft set theory and its applications in various fields are progressing rapidly ([5], [11-12], [18-19]). In [17], Shabir and Naz introduced soft topological spaces. In paper [22], the authors introduced some new concepts in soft topological spaces such as soft point, interior point, interior, continuity, and

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compactness. Based on paper [22], in paper [21], the authors introduced the definitions of L-fuzzifying soft topological spaces and L-fuzzifying soft interior spaces . They showed that LF-**STOP** the category L-fuzzifying soft topological spaces and their continuous mapping, and LF-**SIS** the category L-fuzzifying soft interior spaces and their continuous mappings are isomorphic. The notion of proximity was studied in a fuzzy setting ([7], [16]), in an L-fuzzy setting ([9], [15]) and in a fuzzifying setting ([8]). In [3], the author introduced and studied the notions of L-fuzzifying preproximity, L-fuzzifying preproximilly continuous mappings, L-fuzzifying toplogies and L-fuzzifying continuous mappings. In this paper, the notions of soft preproximity and soft preproximilly continuous mappings in L-fuzzifying soft setting are studied. Some relations between theses spaces and L-fuzzifying soft topological spaces are introduced.

2. Preliminaries

In this paper, let $(L, \leq, \wedge, \vee, ')$ denotes a fuzzy lattice [6], i. e., a completely distributive complete lattice with order-reversing involution $'$, i.e., $(L, \leq, \wedge, \vee, ')$ is a complete lattice, for every $\lambda \in \Lambda$ and for every $A_\lambda \subseteq L$, $\bigwedge\limits_{\lambda\in\Lambda} \bigvee A_\lambda = \bigvee\limits_{\psi\in \prod\limits_{\lambda\in\Lambda} A_\lambda} \bigwedge\limits_{\lambda\in\Lambda} \psi(\lambda)$ and $' : L \to L$ is a mapping such that for every $a, b \in L$, $(a')' = a$ and $a \leq b \Rightarrow b' \leq a'$. The smallest element and the largest element in L will be denoted by $\perp$ and $\top$, respectively. It is well Known that in any poset if $a \leq b$ and $a \neq b$, we write $a < b$.

Let L be a non empty reflexive relational structure and let a, b be elements of L. We say that a is way below b if and only if for every non empty directed subset D of L such that $b \leq \bigvee D$ there exists an element d of L such that $d \in D$ and $a \leq d$ [6]. We introduce $a \ll b$ as synonym of a is way below b. A complete lattice L is completely distributive if and only if $b = \bigvee\{a \in L : a \ll b\}$ for each $b \in L$. For $b \in L$, define $\Downarrow b = \{a \in L : a \ll b\}$. Some properties $\Downarrow$ of can be found in [10, 20]. A complete lattice L is continuous if and only if for every $a \in L, a = \bigvee\{b : b \in \Downarrow a\}[6, 10]$. It is observed that any completely distributive lattice is continuous [6,10]. A lattice L is said to be an order-dense chain [9] if and only if for each $a, b \in L$ and $a < b$, there exists $c \in L$ such that $a < c < b$.

Lemma 1. [6,10]. Let $a, b, a_1, a_2, b_1, b_2 \in L$. Then
(1) if $a_1 \leq a_2, b_1 \leq b_2$ and $a_2 \ll b_1$, then $a_1 \ll b_2$;
(2) $0 \ll a$;
(3) if $a_1 \ll b$ and $a_2 \ll b$, then $a_1 \vee a_2 \ll b$;

(4) if $a \ll b$, then $a \leq b$;
(5) if $\top \not\ll \top$, then $\bigvee\{a \in L : a < \top\} = \top$.

Definition 2. [14]. (1) A soft set on an universe X is a pair (M, E) (here E is a nonempty parameter set), and $M : E \to 2^X$ (the set of all subset of X) is a mapping. The set of all soft sets on X is denoted by $\mathbf{S}(\mathbf{X})$.
(2) For two given subsets (F, A), $(G, B) \in \mathbf{S}(\mathbf{X})$, we say that (F, A) is a soft subset of (G, B), denoted by $(F, A)\widetilde{\subseteq}\ (G, B)$,
(a) $A \subseteq B$;
(b) for all $e \in A$, $F(e) \subseteq G(e)$.
If $(F, A)\widetilde{\subseteq}\ (G, B)$ and $(F, A)\widetilde{\supseteq}(G, B)$, we say (F, A) and (G, B) be soft equal. We denote it by $(F, A) = (G, B)$.

Definition 3. [11]. The union of two soft sets $(F, A)$and (G, B) on X is the soft set (H, C), where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e) & e \in A\backslash B \\ G(e) & e \in B\backslash A \quad (\forall e \in C). \\ F(e) \cup G(e) & e \in A \cap B \end{cases}$$

We write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 4. [17]. The intersection of two soft sets $(F, A)$and (G, B) on X is the soft set (H, C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ $(\forall e \in C)$. We write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 5. [17]. (1) For each $A \in 2^X$, $(\widetilde{A}, E) \in \mathbf{S}(\mathbf{X})$ is defined by $\widetilde{A}(e) = A$ for each $e \in E$;we identify $\widetilde{\{x\}}$ with $\widetilde{x}$ for each $x \in X$. For each $(M, E) \in \mathbf{S}(\mathbf{X})$, $(M^c, E) \in \mathbf{S}(\mathbf{X})$ is defined by $M^c(e) = X\backslash M(e)(\forall e \in E)$; sometimes we use $(M, E)^c$ (resp. $\widetilde{A}$) to replace (M^c, E) (resp. $(\widetilde{A}, E)$).
(2) For a given subset $\{(H_\lambda, E_\lambda)\}_{\lambda\in\Lambda} \subseteq \mathbf{S}(\mathbf{X})$, we call members $(M, E) = \widetilde{\bigcup}_{\lambda\in\Lambda}(H_\lambda, E_\lambda)$ and $(N, E) = \widetilde{\bigcap}_{\lambda\in\Lambda}(H_\lambda, E_\lambda)$ of $\mathbf{S}(\mathbf{X})$ union and intersection of the family $\{(H_\lambda, E_\lambda)\}_{\lambda\in\Lambda}$, respectively, which are defined by $M(e) = \bigcup_{\lambda\in\Lambda} H_\lambda(e)$ $(\forall e \in E_\lambda)$ and $N(e) = \bigcap_{\lambda\in\Lambda} H_\lambda(e)(\forall e \in E_\lambda)$.
(3) For a given subset $(H, E) \in \mathbf{S}(\mathbf{X})$, and $x \in X$, we say that $x \in (H, E)$ whenever $x \in H(e)$ for each $e \in E$. If $x \notin H(e)$ for some $e \in E$, we say $x \notin (H, E)$. (4) For two given subsets $(F, A), (G, B) \in \mathbf{S}(\mathbf{X})$, then
(i) $((F, A)\widetilde{\cup}(G, B))^c = (F, A)^c\widetilde{\cap}\ (G, B)^c$.;
(ii) $((F, A)\widetilde{\cap}(G, B))^c = (F, A)^c\widetilde{\cup}\ (G, B)^c$.

Definition 6. [19]. Defined soft function $(f, g) : \mathbf{S}(\mathbf{X}) \to \mathbf{S}(\mathbf{Y})$ by

$$(f, g)(M, E) = (\overrightarrow{g}(M), f(E))$$

for each $(M,E) \in \mathbf{S}(\mathbf{X})$,$(\overrightarrow{g}(M), f(E)) \in \mathbf{S}(\mathbf{Y})$ and

$$(f,g)^{-1}(N,F) = (\overleftarrow{g} \circ N \circ f, f^{-1}(F))$$

for each $(N,F) \in \mathbf{S}(\mathbf{Y})$, $(\overleftarrow{g} \circ N \circ f, f^{-1}(F)) \in \mathbf{S}(\mathbf{Y})$, where for every $\alpha \in f(E)$ and for every $e \in f^{-1}(F)$ we have

$$\overrightarrow{g}(M)(\alpha) \;\; = \bigcup_{f(e)=\alpha} g(M(e)), (\overleftarrow{g} \circ N \circ f)(e) = \overleftarrow{g}(N(f(e)))$$

$f(E)$ is the image of E in the category **SET**, $f^{-1}(F)$ is the preimage of F in the category **SET.** $\overrightarrow{g}(M)$ is defined by the Zadeh extension principle, $\overleftarrow{g}(M)$ is the backward operator induced by the mapping $g : X \to Y$.

Definition 7. [21]. An L-fuzzifying soft topology on a set X is a mapping $\tau : \mathbf{S}(\mathbf{X}) \to L$ such that
(LFST1) $\tau(\widetilde{\phi}) = \tau(\widetilde{X}) = \top$;
(LFST2) $\forall (F,A), (G,B) \in \mathbf{S}(\mathbf{X})$, $\tau((F,A)\widetilde{\sqcap}(G,B)) \geq \tau(F,A) \wedge \tau(G,B))$;
(LFST3) $\forall \{(F_\lambda, A_\lambda)\}_{\lambda\in\Lambda} \subseteq \mathbf{S}(\mathbf{X})$, $\tau(\widetilde{\bigcup_{\lambda\in\Lambda}} (F_\lambda, A_\lambda)) \geq \bigwedge_{\lambda\in\Lambda} \tau(F_\lambda, A_\lambda)$.
$\tau(F,A)$ can be interpreted as the degree to which (F,A) is an open soft set, the triple (X,τ,E) is called an L-fuzzifying soft topological space. A mapping $g : X \to Y$ from an L-fuzzifying soft topological space (X,τ,E) to another L-fuzzifying soft topological space (Y,σ,E) is said to be continuous if for every $(F,A) \in \mathbf{S}(\mathbf{X})$,

$$\tau((id_E, g)^{-1}(F,A)) \geq \sigma(F,A).$$

Definition 8. [21,22]. (1) The soft set $(M,E) \in \mathbf{S}(\mathbf{X})$ is called a soft point in $\widetilde{X}$, denoted by e_M, if for the element $e \in E, M(e) \neq \phi$ and $M(e_\circ) = \phi$ for all $e_\circ \in E\backslash\{e\}$.
(2) The soft point e_M is said to be in the soft set (N,E), for each $e \in E$, denoted by $e_M \widetilde{\in} (N,E)$,we have $M(e) \subseteq N(e)$.
(3) Let $e_M \widetilde{\in} \widetilde{X}$ and $(N,E) \widetilde{\subseteq} \widetilde{X}$. If $e_M \widetilde{\in} (N,E)$, then $e_M \widetilde{\notin} (N,E)^c$.

$\mathbf{SP}(\mathbf{X})$ denoted the set of all soft points in $\widetilde{X}$. Obviously, if $e_M \widetilde{\in} \mathbf{SP}(\mathbf{X})$, then $(id_E, g)(e_M) \widetilde{\in} \mathbf{SP}(\mathbf{Y})$.

3. L-fuzzifying soft preproximity

Definition 9. Let X be a universe of discourse, $\widetilde{\delta} : \mathbf{S}(\mathbf{X}) \times \mathbf{S}(\mathbf{X}) \to L$ satisfies the following conditions:

$(\widetilde{\delta}1)$ $\widetilde{\delta}(\widetilde{X},\widetilde{\phi})=\widetilde{\delta}(\widetilde{\phi},\widetilde{X})=\perp$;
$(\widetilde{\delta}2)$ if $\widetilde{\delta}((F,A),(G,B))\ll\top$, then $(F,A)\widetilde{\subseteq}(G^c,B)$;
$(\widetilde{\delta}3)$ $\widetilde{\delta}((F,A)\widetilde{\cup}(G,B),(H,C))=\widetilde{\delta}((F,A),(H,C))\vee\widetilde{\delta}((G,B),(H,C))$
and $\widetilde{\delta}((F,A),(G,B)\widetilde{\cup}(H,C))=\widetilde{\delta}((F,A),(G,B))\vee\widetilde{\delta}((F,A),(H,C))$.
Then $\widetilde{\delta}$ is called an L-fuzzifying soft preproximity on X and $(X,\widetilde{\delta})$ is an L-fuzzifying soft preproximity space.

Lemma 10. If $(F,A)\widetilde{\subseteq}(G,B)$, then

$$\widetilde{\delta}((F,A),(H,C))\le\widetilde{\delta}((G,B),(H,C))$$
$$\widetilde{\delta}((H,C),(F,A))\le\widetilde{\delta}((H,C),(G,B)).$$

Proof. Since $(F,A)\widetilde{\subseteq}(G,B),(F,A)\widetilde{\cup}(G,B)=(G,B)$. Then from $(\widetilde{\delta}3)$ we have

$$\begin{aligned}&\widetilde{\delta}((F,A),(H,C))\vee\widetilde{\delta}((G,B),(H,C))\\&=\widetilde{\delta}((F,A)\widetilde{\cup}(G,B),(H,C))=\widetilde{\delta}((G,B),(H,C)).\end{aligned}$$

Therefore $\widetilde{\delta}((F,A),(H,C))\le\widetilde{\delta}((G,B),(H,C))$. Similarly, $\widetilde{\delta}((H,C),(F,A))\le\widetilde{\delta}((H,C),(G,B))$.

Theorem 11. *Let $(X,\widetilde{\delta})$ be an L-fuzzifying soft preproximity space. The mapping $I_{\widetilde{\delta}}:\mathbf{S}(\mathbf{X})\times L\backslash\{\top\}\to\mathbf{S}(\mathbf{X})$ defined by*

$$I_{\widetilde{\delta}}((F,A),a)=\widetilde{\bigcup_{(H,C)\in\mathbf{S}(\mathbf{X}),\widetilde{\delta}((H,C),(F,A)^c)\ \ll\ a'}}(H,C)$$

for every $(F,A)\in\mathbf{S}(\mathbf{X})$,$a\in L\backslash\{\top\}$ has the following properties:
(1) $I_{\widetilde{\delta}}(\widetilde{X},a)=\widetilde{X}$;
(2) $I_{\widetilde{\delta}}((F,A),a)\widetilde{\subseteq}(F,A)$;
(3) *If* $(F,A)\widetilde{\subseteq}(G,B)$, *then* $I_{\widetilde{\delta}}((F,A),a)\widetilde{\subseteq}I_{\widetilde{\delta}}((G,B),a)$;
(4)*If* $b'\le a'$, *then* $I_{\widetilde{\delta}}((F,A),b)\widetilde{\subseteq}I_{\widetilde{\delta}}((F,A),a)$;
(5) $I_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B),a)=I_{\widetilde{\delta}}((F,A),a)\widetilde{\cap}I_{\widetilde{\delta}}((G,B),a)$.

Proof. (1) Since $\perp\ll a'$ for every $a\in L\backslash\{\top\}$ and $\widetilde{\delta}(\widetilde{X},\widetilde{X}^c)=\widetilde{\delta}(\widetilde{X},\widetilde{\phi})=\perp$, $I_{\widetilde{\delta}}(\widetilde{X},a)=\widetilde{X}$.
(2) Since $\widetilde{\delta}((H,C),(F,A)^c)\le\widetilde{\delta}((H,C),(F,A)^c),a'\le\top$ and $\widetilde{\delta}((H,C),(F,A)^c)\ll a'$, then from Lemma 1(1), $\widetilde{\delta}((H,C),(F,A)^c)\ll\top$. So from $(\widetilde{\delta}2)$, $(H,C)\widetilde{\subseteq}[(F,A)^c]^c=(F,A)$. Therefore $I_{\widetilde{\delta}}((F,A),a)\widetilde{\subseteq}(F,A)$.
(3) Suppose that $(F,A)\widetilde{\subseteq}(G,B)$. Then $(G,B)^c\widetilde{\subseteq}(F,A)^c$ and from Lemma 10 we have $\widetilde{\delta}((H,C),(G,B)^c)\le\widetilde{\delta}((H,C),(F,A)^c)$. So, if $\widetilde{\delta}((H,C),(F,A)^c)\ll a'$, then

from Lemma 1.1(1) we have $\widetilde{\delta}((H,C),(G,B)^c) \ll a'$. Therefore $I_{\widetilde{\delta}}((F,A),a)\widetilde{\subseteq}I_{\widetilde{\delta}}((G,B),a)$.
(4) Suppose $\widetilde{\delta}((H,C),(F,A)^c) \ll b'$. Then from Lemma 1 (1) we have $\widetilde{\delta}((H,C),(F,A)^c) \ll a'$. Therefore $I_{\widetilde{\delta}}((F,A),b)\widetilde{\subseteq}I_{\widetilde{\delta}}((F,A),a)$.
(5) From (3) we have $I_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B),a)\widetilde{\subseteq}I_{\widetilde{\delta}}((F,A),a)\widetilde{\cap}I_{\widetilde{\delta}}((G,B),a)$.
Let $e_M \in I_{\widetilde{\delta}}((F,A),a)\widetilde{\cap}I_{\widetilde{\delta}}((G,B),a)$. Thus there exist $(H,C_1),(K,C_2) \in \mathbf{S}(\mathbf{X})$ such that

$$\widetilde{\delta}((H,C_1),(F,A)^c) \ll a', \ \ \widetilde{\delta}((K,C_2),(G,B)^c) \ll a'$$

and $e_M \in (H,C_1)\widetilde{\cap}(K,C_2)$. From Lemmas 10 and 1(1),(3) and ($\widetilde{\delta}3$) we have

$$\begin{aligned}
&\widetilde{\delta}((H,C_1)\widetilde{\cap}(K,C_2),(F,A)^c\widetilde{\cup}(G,B)^c) \\
&= \widetilde{\delta}((H,C_1)\widetilde{\cap}(K,C_2),(F,A)^c) \vee \widetilde{\delta}((H,C_1)\widetilde{\cap}(K,C_2),(G,B)^c) \\
&\le \widetilde{\delta}((H,C_1),(F,A)^c) \vee \widetilde{\delta}((K,C_2),(G,B)^c) \ll a'.
\end{aligned}$$

Thus

$$(H,C_1)\widetilde{\cap}(K,C_2)\widetilde{\subseteq}I_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B),a).$$

Therefore $e_M \in I_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B),a)$. Hence

$$I_{\widetilde{\delta}}((F,A),a)\widetilde{\cap}I_{\widetilde{\delta}}((G,B),a)\widetilde{\subseteq}I_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B),a).$$

Theorem 12. *Let $(X,\widetilde{\delta})$ be an L-fuzzifying soft preproximity space and $\top\not\ll\top$. Then the mapping $\tau^1_{\widetilde{\delta}} : \mathbf{S}(\mathbf{X}) \to L$ defined by*

$$\tau^1_{\widetilde{\delta}}((F,A)) = \bigvee_{a\in L\setminus\{\top\}, I_{\widetilde{\delta}}((F,A),a)=(F,A)} a$$

is an L-fuzzifying soft topology on X.

Proof. (1) Applying Theorem 11 (1) and Lemma 1 (5) we have

$$\tau^1_{\widetilde{\delta}}(\widetilde{X}) = \bigvee_{a\in L\setminus\{\top\}, I_{\widetilde{\delta}}(\widetilde{X},a)=\widetilde{X}} a = \top.$$

We have from Theorem 11 (2) that $\widetilde{\phi}\widetilde{\subseteq}I_{\widetilde{\delta}}(\widetilde{\phi},a)\widetilde{\subseteq}\widetilde{\phi}$ and applying Lemma 1 (5) we obtain $\tau^1_{\widetilde{\delta}}(\widetilde{\phi}) = \top$.

(2) Applying Theorem 11 we obtain $I_{\widetilde{\delta}}((F,A),a)=(F,A)$ and $I_{\widetilde{\delta}}((G,B),b)=(G,B)$ implies $I_{\widetilde{\delta}}((F,A),a\wedge b)=(F,A)$ and $I_{\widetilde{\delta}}((G,B),a\wedge b)=(G,B)$. Thus

$$\begin{aligned}&I_{\widetilde{\delta}}((F,A)\widetilde{\sqcap}(G,B),a\wedge b)\\&=I_{\widetilde{\delta}}((F,A),a\wedge b)\wedge I_{\widetilde{\delta}}((G,B),a\wedge b)=(F,A)\widetilde{\sqcap}(G,B).\end{aligned}$$

Therefore

$$\begin{aligned}&\tau^1_{\widetilde{\delta}}((F,A))\wedge\tau^1_{\widetilde{\delta}}((G,B))\\&=\left(\bigvee_{a\in L\backslash\{\top\},I_{\widetilde{\delta}}((F,A),a)=(F,A)}a\right)\wedge\left(\bigvee_{b\in L\backslash\{\top\},I_{\widetilde{\delta}}((G,B),b)=(G,B)}b\right)\\&=\bigvee_{(a\wedge b)\in L\backslash\{\top\},I_{\widetilde{\delta}}((F,A),a)=(F,A),I_{\widetilde{\delta}}((G,B),b)=(G,B)}(a\wedge b)\\&\leq\bigvee_{(a\wedge b)\in L\backslash\{\top\},I_{\widetilde{\delta}}((F,A)\widetilde{\sqcap}(G,B),a\wedge b)=(F,A)\widetilde{\sqcap}(G,B)}(a\wedge b)\\&\leq\bigvee_{c\in L\backslash\{\top\},I_{\widetilde{\delta}}((F,A)\widetilde{\sqcap}(G,B),c)=(F,A)\widetilde{\sqcap}(G,B)}c\\&=\tau^1_{\widetilde{\delta}}((F,A)\widetilde{\sqcap}(G,B)).\end{aligned}$$

(3) Suppose that $I_{\widetilde{\delta}}((F_\lambda,A_\lambda),a_\lambda)=(F_\lambda,A_\lambda),\forall\lambda\in\Lambda$. Then From Theorem 11 (4) we have

$$(F_\lambda,A_\lambda)\widetilde{\subseteq}I_{\widetilde{\delta}}((F_\lambda,A_\lambda),\bigwedge_{\lambda\in\Lambda}a_\lambda),\forall\lambda\in\Lambda.$$

Applying Theorem 11 (2) and (3) we have

$$\begin{aligned}&(F_\lambda,A_\lambda)\widetilde{\subseteq}I_{\widetilde{\delta}}((F_\lambda,A_\lambda),\bigwedge_{\lambda\in\Lambda}a_\lambda)\\&\widetilde{\subseteq}I_{\widetilde{\delta}}(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda),\bigwedge_{\lambda\in\Lambda}a_\lambda)\widetilde{\subseteq}\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda).\end{aligned}$$

Therefore $I_{\widetilde{\delta}}(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda),\bigwedge_{\lambda\in\Lambda}a_\lambda)=\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)$.

Thus

$$\bigwedge_{\lambda\in\Lambda}\tau^1_{\widetilde{\delta}}((F_\lambda,A_\lambda))=\bigwedge_{\lambda\in\Lambda}\bigvee_{d_\lambda\in L\backslash\{\top\},I_{\widetilde{\delta}}((F_\lambda,A_\lambda),d_\lambda)=(F_\lambda,A_\lambda)}d_\lambda.$$

Therefore from completely distributive law we have

$$\begin{aligned}&\bigwedge_{\lambda\in\Lambda}\tau^1_{\widetilde{\delta}}((F_\lambda,A_\lambda))=\bigwedge_{\lambda\in\Lambda}\bigvee_{d_\lambda\in L\backslash\{\top\},I_{\widetilde{\delta}}((F_\lambda,A_\lambda),d_\lambda)=(F_\lambda,A_\lambda)}d_\lambda\\&=\bigvee_{f\in\prod_{\lambda\in\Lambda}d_\lambda}\bigwedge_{\lambda\in\Lambda}f(\lambda)\leq\bigvee_{d\in L\backslash\{\top\},I_{\widetilde{\delta}}(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda),d)=\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)}d\\&=\tau^1_{\widetilde{\delta}}(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)).\end{aligned}$$

Theorem 13. *Let* $(X,\widetilde{\delta})$ *be an L-fuzzifying soft preproximity space. Then the mapping* $\tau^2_{\widetilde{\delta}} : \mathbf{S}(\mathbf{X}) \to L$ *defined by*

$$\tau^2_{\widetilde{\delta}}((F,A)) = \bigwedge_{e_M \in (F,A)} \left(\widetilde{\delta}(\{e_M\},(F,A)^c)\right)'$$

is an L-fuzzifying soft topology on X.

Proof. (1) From $(\widetilde{\delta}1)$ we have

$$\tau^2_{\widetilde{\delta}}(\widetilde{X}) = \bigwedge_{e_M \in \widetilde{X}} \left(\widetilde{\delta}(\{e_M\},\widetilde{\phi})\right)' = \bot' = \top$$
$$\tau^2_{\widetilde{\delta}}(\widetilde{\phi}) = \bigwedge_{e_M \in \widetilde{\phi}} \left(\widetilde{\delta}(\{e_M\},\widetilde{X})\right)' = \bot' = \top.$$

(2) From $(\widetilde{\delta}3)$ we have

$$\begin{aligned}
&\tau^2_{\widetilde{\delta}}((F,A)\widetilde{\cap}(G,B)) = \bigwedge_{e_M \in (F,A)\widetilde{\cap}(G,B)} \left(\widetilde{\delta}(\{e_M\},((F,A)\widetilde{\cap}(G,B))^c)\right)' \\
&= \bigwedge_{e_M \in (F,A)\widetilde{\cap}(G,B)} \left(\widetilde{\delta}(\{e_M\},(F,A)^c\widetilde{\cup}(G,B)^c)\right)' \\
&= \bigwedge_{e_M \in (F,A)\widetilde{\cap}(G,B)} (\widetilde{\delta}(\{e_M\},(F,A)^c) \vee \widetilde{\delta}(\{e_M\},(G,B)^c))' \\
&= \bigwedge_{e_M \in (F,A)\widetilde{\cap}(G,B)} \left(\left(\widetilde{\delta}(\{e_M\},(F,A)^c)\right)' \wedge \left(\widetilde{\delta}(\{e_M\},(G,B)^c)\right)' \right) \\
&\geq \bigwedge_{e_M \in (F,A)} \left(\widetilde{\delta}(\{e_M\},(F,A)^c)\right)' \wedge \bigwedge_{e_M \in (G,B)} \left(\widetilde{\delta}(\{e_M\},(G,B)^c)\right)' \\
&= \tau^2_{\widetilde{\delta}}((F,A)) \wedge \tau^2_{\widetilde{\delta}}((G,B)).
\end{aligned}$$

(3) From Lemma 10 we have

$$\begin{aligned}
&\tau^2_{\widetilde{\delta}}\left(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)\right) = \bigwedge_{e_M \in \widetilde{\bigcup}_{\lambda\in\Lambda}(F_\lambda,A_\lambda)} \left(\widetilde{\delta}\left(\{e_M\}, \widetilde{\bigcap_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)^c\right)\right)' \\
&= \bigwedge_{\lambda\in\Lambda} \bigwedge_{e_M \in (F_\lambda,A_\lambda)} \left(\widetilde{\delta}\left(\{e_M\}, \widetilde{\bigcap_{\lambda\in\Lambda}}(F_\lambda,A_\lambda)^c\right)\right)' \\
&\geq \bigwedge_{\lambda\in\Lambda} \bigwedge_{e_M \in (F_\lambda,A_\lambda)} \left(\widetilde{\delta}\left(\{e_M\},(F_\lambda,A_\lambda)^c\right)\right)' \\
&= \bigwedge_{\lambda\in\Lambda} \tau^2_{\widetilde{\delta}}((F_\lambda,A_\lambda)).
\end{aligned}$$

Definition 14. An L-fuzzifying soft preproximity is said to be principal provided that:
$(\widetilde{\delta}3)$ $\widetilde{\delta}\left(\widetilde{\bigcup_{\lambda\in\Lambda}}(F_\lambda, A_\lambda),(G,B)\right) \leq \bigvee_{\lambda\in\Lambda}\widetilde{\delta}\left((F_\lambda, A_\lambda),(G,B)\right).$

Theorem 15. *Let $(X,\widetilde{\delta})$ be a principal L-fuzzifying soft preproximity space. Then the mapping $\tau^2_{\widetilde{\delta}}:\mathbf{S}(\mathbf{X})\to L$ defined by*

$$\tau^2_{\widetilde{\delta}}((F,A)) = \left(\widetilde{\delta}((F,A),(F,A)^c)\right)'.$$

Proof.

$$\begin{aligned}\tau^2_{\widetilde{\delta}}((F,A)) &= \bigwedge_{e_M\in(F,A)}\left(\widetilde{\delta}(\{e_M\},(F,A)^c)\right)' \\ &= \left(\bigvee_{e_M\in(F,A)}\widetilde{\delta}(\{e_M\},(F,A)^c)\right)' \\ &= \left(\widetilde{\delta}(\bigcup_{e_M\in(F,A)}\{e_M\},(F,A)^c)\right)' = \left(\widetilde{\delta}((F,A),(F,A)^c)\right)'.\end{aligned}$$

Theorem 16. *Let $\tau:\mathbf{S}(\mathbf{X})\to L$ be an L-fuzzifying soft topology on X. Define the mapping $\widetilde{\delta}_\tau:\mathbf{S}(\mathbf{X})\times\mathbf{S}(\mathbf{X})\to L$ as follows:*

$$\widetilde{\delta}_\tau((F,A),(G,B)) = \begin{cases}(\bigvee_{(H,C)\in\Phi((F,A),(G,B))}\tau((H,C)))', & \Phi((F,A),(G,B))\neq\widetilde{\phi} \\ \top, & \Phi((F,A),(G,B))=\widetilde{\phi}\end{cases}$$

where $\Phi:\mathbf{S}(\mathbf{X})\times\mathbf{S}(\mathbf{X})\to 2^{\mathbf{S}(\mathbf{X})}$ is defined as follows:

$$\Phi((F,A),(G,B)) = \{(H,C)\in\mathbf{S}(\mathbf{X}) : (F,A)\widetilde{\subseteq}(H,C)\widetilde{\subseteq}(G,B)^c\}.$$

If $\top\not\ll\top$, then the mapping $\widetilde{\delta}_\tau$ is an L-fuzzifying soft preproximity on X.

Proof. $(\widetilde{\delta}_\tau 1)$ Since $\Phi(\widetilde{X},\widetilde{\phi}) = \{\widetilde{X}\}, \widetilde{\delta}_\tau(\widetilde{X},\widetilde{\phi}) = \top' = \bot$. Also, since $\Phi(\widetilde{\phi},\widetilde{X}) = \{\widetilde{\phi}\}, \widetilde{\delta}_\tau(\widetilde{\phi},\widetilde{X}) = \top' = \bot$.
$(\widetilde{\delta}_\tau 2)$ Suppose $\widetilde{\delta}_\tau((F,A),(G,B))\ll\top$. Then

$$\widetilde{\delta}_\tau((F,A),(G,B))\neq\top.$$

So, $\widetilde{\delta}_\tau((F,A),(G,B)) < \top$. Therefore $\Phi((F,A),(G,B)) \neq \widetilde{\phi}$, i.e., there exists $(H,C) \in \mathbf{S}(\mathbf{X})$ such that

$$(F,A)\widetilde{\subseteq}(H,C)\widetilde{\subseteq}(G,B)^c.$$

$(\widetilde{\delta}_\tau 3)$ Suppose $(H,C)\widetilde{\subseteq}(G,B)$. If $\Phi((F,A),(G,B)) = \phi$, then $\widetilde{\delta}_\tau((F,A),(G,B)) = \top \geq \widetilde{\delta}_\tau((F,A),(H,C))$. If $\Phi((F,A),(G,B)) \neq \phi$, then $\Phi((F,A),(H,C)) \neq \phi$. Since $\Phi((F,A),(G,B))\widetilde{\subseteq}\Phi((F,A),(H,C))$, then we have

$$\begin{aligned}
&\widetilde{\delta}_\tau((F,A),(H,C)) = \left(\bigvee_{(L,D)\in\Phi((F,A),(H,C))} \tau((L,D))\right)' \\
&= \left(\bigvee_{(L,D)\in\Phi((F,A),(H,C))} \tau((L,D))\right)' \leq \left(\bigvee_{(M,E)\in\Phi((F,A),(G,B))} \tau((M,E))\right)' \\
&= \widetilde{\delta}_\tau((F,A),(G,B)).
\end{aligned}$$

Therefore

$$\begin{aligned}
&\widetilde{\delta}_\tau((F,A),(G_1,B_1)) \leq \widetilde{\delta}_\tau((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)) \\
&\widetilde{\delta}_\tau((F,A),(G_2,B_2)) \leq \widetilde{\delta}_\tau((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)).
\end{aligned}$$

Hence

$$\begin{aligned}
&\widetilde{\delta}_\tau((F,A),(G_1,B_1)) \vee \widetilde{\delta}_\tau((F,A),(G_2,B_2)) \\
&\leq \widetilde{\delta}_\tau((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)).
\end{aligned}$$

If $(L,D) \in \Phi((F,A),(G_1,B_1))$ and $(M,E) \in \Phi((F,A),(G_2,B_2))$, one can deduce that

$$(L,D)\widetilde{\cap}(M,E) \in \Phi((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)).$$

Thus we have

$$
\begin{aligned}
&\widetilde{\delta}_\tau((F,A),(G_1,B_1)) \vee \widetilde{\delta}_\tau((F,A),(G_2,B_2))\\
&= \left(\bigvee_{(L,D)\in\Phi((F,A),(G_1,B_1))} \tau((L,D)) \right)' \vee \left(\bigvee_{(M,E)\in\Phi((F,A),(G_2,B_2))} \tau((M,E)) \right)'\\
&= \Big(\left(\bigvee_{(L,D)\in\Phi((F,A),(G_1,B_1))} \tau((L,D)) \right) \wedge \left(\bigvee_{(M,E)\in\Phi((F,A),(G_2,B_2))} \tau((M,E)) \right) \Big)'\\
&= \Big(\bigvee_{(L,D)\in\Phi((F,A),(G_1,B_1)),(M,E)\in\Phi((F,A),(G_2,B_2))} (\tau((L,D)) \wedge \tau((M,E))) \Big)'\\
&\geq \Big(\bigvee_{(L,D)\in\Phi((F,A),(G_1,B_1)),(M,E)\in\Phi((F,A),(G_2,B_2))} \big(\tau((L,D)\widetilde{\cap}(M,E))\big) \Big)'\\
&\geq \Big(\bigvee_{(L,D)\widetilde{\cap}(M,E)\in\Phi((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)))} \big(\tau((L,D)\widetilde{\cap}(M,E))\big) \Big)'\\
&\geq \left(\bigvee_{(F_\circ,A_\circ)\in\Phi((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2))} \tau((F_\circ,A_\circ)) \right)'\\
&= \widetilde{\delta}_\tau((F,A),(G_1,B_1)\widetilde{\cup}(G_2,B_2)).
\end{aligned}
$$

For the second assertion of $(\widetilde{\delta}_\tau 3)$ Suppose $(H,C)\widetilde{\subseteq}(G,B)$. If $\Phi((G,B),(F,A)) = \phi$, then

$$\widetilde{\delta}_\tau((G,B),(F,A)) = \top \geq \widetilde{\delta}_\tau((H,C),(F,A)).$$

Let $(L,D) \in \Phi((G,B),(F,A))$. Then $(H,C)\widetilde{\subseteq}(L,D)\widetilde{\subseteq}(F,A)^c$. So $(L,D) \in \Phi((H,C),(F,A))$. Thus

$$
\begin{aligned}
&\Phi((G,B),(F,A))\widetilde{\subseteq}\Phi((H,C),(F,A)),\\
&\widetilde{\delta}_\tau((H,C),(F,A)) = \left(\bigvee_{(L,D)\in\Phi((H,C),(F,A))} \tau((L,D)) \right)'\\
&= \left(\bigvee_{(L,D)\in\Phi((H,C),(F,A))} \tau((L,D)) \right)'\\
&\geq \left(\bigvee_{(M,E)\in\Phi((G,B),(F,A))} \tau((M,E)) \right)'\\
&= \widetilde{\delta}_\tau((G,B),(F,A)).
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&\widetilde{\delta}_\tau((G_1,B_1),(F,A)) \leq \widetilde{\delta}_\tau((G_1,B_1)\widetilde{\cup}(G_2,B_2),(F,A))\\
&\widetilde{\delta}_\tau((G_2,B_2),(F,A)) \leq \widetilde{\delta}_\tau((G_1,B_1)\widetilde{\cup}(G_2,B_2),(F,A)).
\end{aligned}
$$

Hence $\widetilde{\delta}_\tau((G_1,B_1),(F,A))\vee\widetilde{\delta}_\tau((G_2,B_2),(F,A))\le\widetilde{\delta}_\tau((G_1,B_1)\widetilde{\cup}(G_2,B_2),(F,A))$. Now

$$\begin{aligned}
&\widetilde{\delta}_\tau((G_1,B_1),(F,A))\vee\widetilde{\delta}_\tau((G_2,B_2),(F,A))\\
&=\Bigg(\bigvee_{(L,D)\in\Phi((G_1,B_1),(F,A))}\tau((L,D))\Bigg)'\vee\Bigg(\bigvee_{(M,E)\in\Phi((G_2,B_2),(F,A))}\tau((M,E))\Bigg)'\\
&=\Bigg(\Bigg(\bigvee_{(L,D)\in\Phi((G_1,B_1),(F,A))}\tau((L,D))\Bigg)\wedge\Bigg(\bigvee_{(M,E)\in\Phi((G_2,B_2),(F,A))}\tau((M,E))\Bigg)\Bigg)'\\
&=\Bigg(\bigvee_{(L,D)\in\Phi((G_1,B_1),(F,A)),(M,E)\in\Phi((G_2,B_2),(F,A))}(\tau((L,D))\wedge\tau((M,E)))\Bigg)'\\
&\ge\Bigg(\bigvee_{(L,D)\in\Phi((G_1,B_1),(F,A)),(M,E)\in\Phi((G_2,B_2),(F,A))}\big(\tau((L,D)\widetilde{\cup}(M,E)))\Bigg)'\\
&\ge\Bigg(\bigvee_{(L,D)\widetilde{\cup}(M,E)\in\Phi((F,A),(H_1,B_1)\widetilde{\cup}(H_2,B_2)))}\big(\tau((L,D)\widetilde{\cup}(M,E)))\Bigg)'\\
&\ge\Bigg(\bigvee_{(F_\circ,A_\circ)\in\Phi((G_1,B_1)\widetilde{\cup}(G_2,B_2),(F,A))}\tau((F_\circ,A_\circ))\Bigg)'\\
&=\widetilde{\delta}_\tau((G_1,B_1)\widetilde{\cup}(G_2,B_2),(F,A)).
\end{aligned}$$

Definition 17. Let $(X,\widetilde{\delta}_1)$ and $(Y,\widetilde{\delta}_2)$ be two L-fuzzifying soft preproximity spaces. A mapping $f:(X,\widetilde{\delta}_1)\to(Y,\widetilde{\delta}_2)$ is said to be L-fuzzifying soft preproximilly continuous if $\widetilde{\delta}_1(f^{-1}((F,A)),f^{-1}((G,B)))\le\widetilde{\delta}_2((F,A),(G,B))$, for any $(F,A),(G,B)\in 2^{\mathbf{S(Y)}}$.

Lemma 18. Let $f:(X,\widetilde{\delta}_1)\to(Y,\widetilde{\delta}_2)$ be L-fuzzifying soft preproximilly continuous mapping. If $I_{\widetilde{\delta}_2}((G,B),a)=(G,B)$, then $I_{\widetilde{\delta}_1}(f^{-1}((G,B)),a)=f^{-1}((G,B))$.

Proof. From Lemma 1 (1) we have

$$\begin{aligned}
&I_{\widetilde{\delta}_1}(f^{-1}((G,B)),a)\widetilde{\subseteq}f^{-1}((G,B))=f^{-1}(I_{\widetilde{\delta}_2}((G,B),a))\\
&=f^{-1}\Bigg(\widetilde{\bigcup}_{\widetilde{\delta}_2((H,C),(G,B)^c)\ \ll\ a'}(H,C)\Bigg)\\
&\widetilde{\subseteq}\Bigg(\widetilde{\bigcup}_{\widetilde{\delta}_1(f^{-1}((H,C)),f^{-1}((G,B)^c))\ \ll\ a'}f^{-1}((H,C))\Bigg)\\
&\widetilde{\subseteq}\Bigg(\widetilde{\bigcup}_{\widetilde{\delta}_1((L,D),f^{-1}((G,B)^c))\ \ll\ a'}(L,D)\Bigg)\\
&=I_{\widetilde{\delta}_1}(f^{-1}((G,B)),a).
\end{aligned}$$

Theorem 19. *If the mapping* $f : (X, \widetilde{\delta_1}) \to (Y, \widetilde{\delta_2})$ *is an L-fuzzifying soft preproximilly continuous, then the mapping* $f : (X, \tau^1_{\widetilde{\delta}_1}) \to (Y, \tau^1_{\widetilde{\delta}_2})$ *is an L-fuzzifying soft continuous.*

Proof. Let $(G, B) \in 2^{\mathbf{S(Y)}}$. Then

$$\tau^1_{\widetilde{\delta}_1}(f^{-1}((G,B))) = \bigvee_{a \in L\setminus\{\top\}, I_{\widetilde{\delta}_1}(f^{-1}((G,B)),a)=f^{-1}((G,B))} a$$
$$\geq \bigvee_{a \in L\setminus\{\top\}, I_{\widetilde{\delta}_2}((G,B),a)=(G,B)} a = \tau^1_{\widetilde{\delta}_2}((G,B)).$$

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