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FOUNDATIONS

Topological approach to generalized soft rough sets via near concepts

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Abstract

An approximation space plays a vital role in the accuracy of approximations of a subset of the universal set. The main goal of this paper is to develop new kinds of soft rough sets models by using the concept of near open sets, where the accuracy of approximations is enhanced significantly. Firstly, the concepts of near soft rough approximations, denoted by "J_{SR}-approximations" for each $J \in \{P, S, \gamma, \alpha, \beta\}$ are proposed as a generalization some previously introduced notions. Then, their properties and relationships are disclosed. Comparisons among the proposed methods and the previous one are obtained. An algorithm has been given for decision-making problems. The proposed algorithm is tested on hypothetical data for the purpose of comparison with already existing methods.

Keywords Soft sets · Soft rough sets · Topological soft rough sets · Near soft rough sets · Accuracy measure

1 Introduction

Soft sets theory is a modern, non-statistical approach to deal with uncertainty and vagueness, proposed by Molodtsov (1999). This theory presents a logical and comprehensible view, to deal with vagueness and uncertainty in the data collected from real-life situations. There are many applications of soft sets ranging from algebra to decision-making problems (Ali 2011). There are many different types of operations available in soft sets, which make it possible to manipulate data without any loss of useful information (Ali et al. 2013). It is not a rival to its contemporary theories, such as fuzzy sets, rough sets, and intuitionistic fuzzy sets, rather it supplements them. Therefore, there exist many hybrid models such as "soft rough sets, fuzzy soft sets and soft topological space." For more details, please see Alcantud (2020), Ali (2011), Ali et al. (2009, 2013, 2017, 2019), Ayub et al. (2020), Babitha and Sunil (2016), Bakeir et al. (2018), El-Bably and El Atik (2021), El-Sayed et al. (2020), El-Sayed and El-Bably (2017), Feng et al. (2011), Li and Xie (2014), Ma et al. (2018) and Maji et al. (2002, 2003).

Rough set theory is another non-statistical model presented by Pawlak (1982) which handles uncertainty by at least two different types of approximations. In literature, many extensions of this theory have been discussed (Abo Khadra et al. 2007; Abo-Tabl 2011, 2013; Ali et al. 2013; Al-shami et al. 2021; Amer et al. 2017; El-Bably and Abo-Tabl 2021; El Sayed et al. 2021; El-Bably 2015; Nawar et al. 2020). The concept of soft rough sets is a hybridization of the soft sets with the rough sets, proposed by Feng et al. (2011). This model has many applications in many real-life problems and it represents a different approach than classical rough sets provided by Pawlak. In fact, they introduce a new notion for approximating the sets using the soft sets. But they have to put some conditions in their approach to satisfy some properties similar to rough approximations. During recent years, many different types of soft rough set models have been proposed for details see Ali (2011), Ali et al. (2009), Ayub et al. (2020), El-Bably and El Atik (2021), El-Sayed et al. (2020) and Shabir et al. (2013).

Rough sets theory and its extensions have very strong relationship with topology. Notions of interior and closure

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in topology are counterparts of lower and upper approximations in rough sets (Pawlak 1982). In rough sets, definable sets give rise to certain type of topologies where every open set is closed as well (Ali et al. 2013).

Recently, the relationship between soft sets and topology has been discussed in Alcantud (2020) and El-Bably et al. (2021). The notion of soft subbasis has been introduced by El-Bably et al. (2021) to propose the notion of "Topological soft rough sets" using a general topology generated from the soft set. By this approach, new types of lower and upper approximations of a subset are obtained, which are generally different from those introduced in Feng et al. (2011). In fact, the introduced approximations satisfy many properties similar to classical Pawlak's (1982) rough set theory without adding any restrictions or conditions. Accordingly, proposed methods open ways for more topological applications in the soft rough sets theory. The basic technique introduced in the current paper provides generalized approximations of subsets based on a new class of nearly open sets studied in Abd El-Monsef et al. (1983), Andrijevi (1986, 1996), El-Bably (2015), Levine (1963), Mashhour et al. (1982), and Njestad (1965). Using the concepts of nearly or near open sets, near soft rough sets, denoted by "J_{SR}-near sets," are introduced and thus new generalized approximations called near soft rough approximations, denoted by "J_{SR}-approximations" for each $J \in \{P, S, \gamma, \alpha, \beta\}$ are obtained. These approximations are more accurate, because here the region of soft boundary is somewhat reduced as compared with already existing approaches. The results and the properties which do not hold in the model given by Feng et al. (2011) have no difficulty in the proposed technique. There is no need to impose any extra conditions here. A Comparison of the suggested model with already existing techniques has been made (Feng et al. 2011; El-Bably et al. 2021). By employing the degree of accuracy of the approximations, we generate six different equivalence relations among the subsets of any finite set. Thus, this equivalence relation classifies the above-mentioned subsets into different classes. These equivalence classes maintain a strict order among them.

Multi-attribute decision making (MADM) is a crucial topic in decision-making theory. MADM's main purpose is to assess the performance of different options in multi-attribute environments. To create an evaluation matrix, a decision maker estimates the performance of each alternative based on a set of attributes. Many decision-making models have been proposed to help decision makers, to reach a reasonable decision. These models are typically built on traditional two-way decision-making (2-WDM) platforms. Yao (2010) recently proposed a new theory called three-way decision making (3-WDM), by introducing a third delayed decision option, (3-WDM) can effectively reduce decision risks. The emergence of 3-WDM opens up a new path and provides new opportunities for MADM study. Zhan et al. (2021a) have introducing a novel three-way decision model based on utility theory in incomplete fuzzy decision systems. Zhan et al. (2021b), 3-WDM model has been employed with the help of outranking relation. Zhan et al. also proposed strategies to design a new 3-WDM model for MADM in Zhan et al. (2020). The literature on multi-criteria decision making and its applications is very rich, for further studies following can be seen Deng et al. (2021), El-Bably and Abo-Tabl (2021), El-Sayed et al. (2020); El Sayed et al. (2021), Ma et al. (2018), Maji et al. (2002), Wang et al. (2021), Zhan and Sun (2019), Zhan and Alcantud (2018), Zhan and Davvaz (2016), Zhang and Zhan (2019) and Zhang et al. (2019, 2020). In present paper, a simple medical application of decision making about the diagnosis of diabetes mellitus is discussed (Bakeir et al. 2018). Here, J_{SR}-near rough approximations are employed to reach a decision. Then, an algorithm is given, which may be very useful for decision-making problems for an information system, in terms of J_{SR}-near rough approximations.

2 Preliminaries

In this section, some basic definitions, results and notations (about Pawlak rough sets, soft sets, soft rough sets and topology) are given, which will be used throughout this paper.

2.1 Pawlak rough set theory

In this subsection, some notions related to rough sets theory presented by Pawlak are given Pawlak (1982).

Definition 2.1.1 (Pawlak 1982) Let U be a finite set called universe, and R be an equivalence relation on U, we use U/R to denote the family of all equivalence classes of R and $[x]_R$ to denote an equivalence class in R containing an element $x \in U$. Then, the pair $A_R = (U, R)$ is called an approximation space and for any $X \subseteq U$, we can define the lower and upper approximation of X by $\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}$ and $\overline{R}(X) = \{x \in U : [x]_R \cap X \neq \phi\}$, respectively. According to Pawlak's definition, X is called a rough set if $\overline{R}(X) \neq \underline{R}(X)$.

Definition 2.1.2 (Pawlak 1982) Let $A_R = (U, R)$ be an approximation space and $X \subseteq U$. Then, the "boundary," "positive" and "negative" regions and the "accuracy" of the approximations of $X \subseteq U$ are defined, respectively, by: $BND_R(X) = \overline{R}(X) - \underline{R}(X)$, $POS_R(X) = \underline{R}(X)$, $NEG_R(X) = U - \overline{R}(X)$ and

$$\mu_R(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|} \quad \text{where} \quad \overline{R}(X) \neq \phi$$

Remark 2.1.1 (Pawlak 1982)

(i) If the boundary region of X is empty ($BND_R(X) = \phi$), then X is a definable (or exact) set with respect to R. Otherwise, if $BND_R(X) \neq \phi$ then X is said to be a rough set with respect to R.
(ii) Sometimes the pair $(\underline{R}(X), \overline{R}(X))$ is referred to the rough set of X with respect to R.

Proposition 2.1.1 (Pawlak 1982) *Let ϕ be the empty set and X^c be the complement of $X \subseteq U$. Pawlak's rough sets have the following properties:*

L_1. $\underline{R}(X) = [\overline{R}(X^c)]^c$.
L_2. $\underline{R}(U) = U$.
L_3. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$.
L_4. $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$.
L_5. $X \subseteq Y \Rightarrow \underline{R}(X) \subseteq \underline{R}(Y)$.
L_6. $\underline{R}(\phi) = \phi$.
L_7. $\underline{R}(X) \subseteq X$.
L_8. $X \subseteq \underline{R}(\overline{R}(X))$.
L_9. $\underline{R}(X) = \underline{R}(\underline{R}(X))$.
L_{10}. $\overline{R}(X) \subseteq \underline{R}(\overline{R}(X))$.
U_1. $\overline{R}(X) = [\underline{R}(X^c)]^c$.
U_2. $\overline{R}(\phi) = \phi$.
U_3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$.
U_4. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$.
U_5. $X \subseteq Y \Rightarrow \overline{R}(X) \subseteq \overline{R}(Y)$.
U_6. $\overline{R}(U) = U$.
U_7. $X \subseteq \overline{R}(X)$.
U_8. $\overline{R}(\underline{R}(X)) \subseteq X$.
U_9. $\overline{R}(\overline{R}(X)) = \overline{R}(X)$.
U_{10}. $\overline{R}(\underline{R}(X)) \subseteq \underline{R}(X)$.

2.2 Soft set theory and soft rough sets

In this subsection, some notions pertaining to soft sets and rough soft sets are given.

Definition 2.2.1 (Molodtsov 1999) Let U be an initial universe of objects and E_U (simply denoted by E) the set of certain parameters in relation to the objects in U. Parameters are often attributes, characteristics or properties of the objects in U. A pair (F, A) is called a "soft set" over U, where $A \subseteq E$, $P(U)$ denote the power set of U and F is a mapping given by $F : A \to P(A)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Note that, sometimes a soft set is denoted by F_A and expressed as a set of ordered pairs $F_A = \{(e, F(e)) : e \in A\}$.

Definition 2.2.2 (Feng et al. 2011) Let F_A be a soft set over U. Then, the pair $A_{SR} = (U, F_A)$ is called a soft approximation space. Based on the soft approximation space A_{SR}, we define the "soft A_{SR}-lower and soft A_{SR}-upper" approximations of any subset $X \subseteq U$, respectively, by the following two operations:

$$\underline{Apr}_{SR}(X) = \{u \in U : \exists e \in A, [u \in F(e) \subseteq X]\},$$
$$\overline{Apr}_{SR}(X) = \{u \in U : \exists e \in A, [u \in F(e), F(e) \cap X \neq \phi]\}.$$

In general, we refer to $\underline{Apr}_{SR}(X)$ and $\overline{Apr}_{SR}(X)$ as soft rough approximations of $X \subseteq U$ with respect to A_{SR}. Moreover, the sets

$$POS_{A_{SR}}(X) = \underline{Apr}_{SR}(X),$$
$$NEG_{A_{SR}}(X) = U - \overline{Apr}_{SR}(X), \quad \text{and}$$
$$BND_{A_{SR}}(X) = \overline{Apr}_{SR}(X) - \underline{Apr}_{SR}(X)$$

are called the soft "A_{SR}-positive region, A_{SR}-negative region and A_{SR}-boundary" regions of $X \subseteq U$, respectively. Clearly, if $\overline{Apr}_{SR}(X) = \underline{Apr}_{SR}(X)$, i.e., $BND_{A_{SR}}(X) = \phi$. Then, $X \subseteq U$ is said to be "soft A_{SR}-definable" or "soft A_{SR}-exact" set; otherwise, X is called a "soft A_{SR}-rough" set. Moreover, we can define the accuracy of the approximations as follows:

$$\mu_{A_{SR}}(X) = \frac{|\underline{R}_{SR}(X)|}{|\overline{R}_{SR}(X)|} \quad \text{where} \quad \overline{R}_{SR}(X) \neq \phi$$

$\mu_{A_{SR}}(X)$ is called the "soft A_{SR}-accuracy" of $X \subseteq U$.

Proposition 2.2.1 (Feng et al. 2011) *Let F_A be a soft set over U and $A_{SR} = (U, F_A)$ a soft approximation space. Then, for each $X \subseteq U$:*

$$\underline{Apr}_{SR}(X) = \bigcup_{e \in A}\{F(e) : F(e) \subseteq X\} \text{and}$$
$$\overline{Apr}_{SR}(X) = \bigcup_{e \in A}\{F(e) : F(e) \cap X \neq \phi\}.$$

Remark 2.2.1 According to the results in Feng et al. (2011), the soft rough approximations "$\underline{Apr}_{SR}(X)$ and $\overline{Apr}_{SR}(X)$" satisfy some properties similar to that of Pawlak's rough approximations (L_1, L_2, L_5, L_6, L_8, L_9, U_2, $U_4 - U_6$ and U_9) without adding any restrictions.

2.3 Topological soft rough sets "TSR-sets"

Topological soft rough sets were introduced in El-Bably et al. (2021) as a new way to study approximations in soft sets. In the present subsection, we give some fundamental concepts and results discussed in El-Bably et al. (2021).

Definition 2.3.1 (El-Bably et al. 2021) Let F_A be a soft set over U and $A_{SR} = (U, F_A)$ a soft approximation space. Then, we define the following:

(i) Soft sub-basis S_{F_A}: $S_{F_A} = \{F(e) : \forall e \in A\}$.
(ii) Soft basis $\mathbf{B}_{F_A}$: $\mathbf{B}_{F_A} = \{X \cap Y : (X, Y) \in S_{F_A} \times S_{F_A}\}$.

Definition 2.3.2 (El-Bably et al. 2021) Let F_A be a soft set over U and let $K = \bigcup_{F(e)\in S_{F_A}} F(e)$. Then, the collection T_{SR} defined as $T_{SR} = \cup\{B : B \in \mathbf{B}_{F_A}\}$ is a topology defined on K generated by the soft basis $\mathbf{B}_{F_A}$. This topology may be called topology generated by $F(e)$ and we call it "soft rough topology" (in briefly, SR-topology).

Definition 2.3.3 (El-Bably et al. 2021) Let $A_{SR} = (U, F_A)$ be a soft approximation space and T_{SR} be the SR-topology on U. Then, the triple $A_{T_{SR}} = (U, F_A, T_{SR})$ is called a "Topological soft rough approximation space" (briefly, T_{SR}-approximation space).

Definition 2.3.4 (El-Bably et al. 2021) Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space. Then, for each $X \subseteq U$, we define the topological soft rough approximations "T_{SR}-lower," and "T_{SR}-upper," respectively, by:

$$\underline{T_{SR}}(X) = \cup\{G \in T_{SR} : G \subseteq X\} \quad \text{and}$$
$$\overline{T_{SR}}(X) = \cap\{H \in T_{SR}{}^{c} : X \subseteq H\}.$$

In general, we refer to $\underline{T_{SR}}(X)$ and $\overline{T_{SR}}(X)$ as "topological soft rough approximations" of $X \subseteq U$ with respect to $A_{T_{SR}}$. Clearly, $\underline{T_{SR}}(X)$ and $\overline{T_{SR}}(X)$ represent the interior and closure of the topology T_{SR}, respectively.

Definition 2.3.5 (El-Bably et al. 2021) Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space. Then, for each $X \subseteq U$ we define the "T_{SR}-positive, T_{SR}-negative, T_{SR}-boundary" regions and the "T_{SR}-accuracy" of the T_{SR}-approximations, respectively, by:

$$POS_{T_{SR}}(X) = \underline{T_{SR}}(X),$$
$$NEG_{T_{SR}}(X) = U - \overline{T_{SR}}(X),$$
$$BND_{T_{SR}}(X) = \overline{T_{SR}}(X) - \underline{T_{SR}}(X) \quad \text{and}$$
$$\mu_{T_{SR}}(X) = \begin{cases} \frac{|\underline{T_{SR}}(X)|}{|\overline{T_{SR}}(X)|} & \text{if } X \neq \phi, \\ 1 & \text{otherwise.} \end{cases}$$

Remark 2.3.1 (i) It is clear that $0 \leq \mu_{T_{SR}}(X) \leq 1$, for any $X \subseteq U$.
(ii) If $\overline{T_{SR}}(X) = \underline{T_{SR}}(X)$, then $BND_{T_{SR}}(X) = \phi$ and $\mu_{T_{SR}}(X) = 1$. Thus, $X \subseteq U$ is said to be "T_{SR}-definable" or "T_{SR}-exact" set; otherwise X is called a "T_{SR}-rough" set.

Remark 2.3.2 (i) According to the results in El-Bably et al. (2021), the T_{SR}-rough approximations "$\underline{T_{SR}}$ and $\overline{T_{SR}}$" satisfy many properties similar to that of Pawlak's rough approximations ($L_1 - U_9$) without adding any restrictions.
(ii) The following results give a relationship in approximations of a subset X, when it is approximated in A_{SR} and $A_{T_{SR}}$.

Theorem 2.3.1 (El-Bably et al. 2021) *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then:*

(i) $\underline{Apr}_{SR}(X) \subseteq \underline{T_{SR}}(X)$.
(ii) $\overline{T_{SR}}(X) \subseteq \overline{Apr}_{SR}(X)$.

Corollary 2.3.1 (El-Bably et al. 2021) *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then:*

(i) $BND_{T_{SR}}(X) \subseteq BND_{A_{SR}}(X)$.
(ii) $\mu_{A_{SR}}(X) \leq \mu_{T_{SR}}(X)$.

Corollary 2.3.2 (El-Bably et al. 2021) *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. If X is a soft exact set, then it is a T_{SR}-exact set.*

3 Near concepts in the topological soft rough approximation space

Topology Kelly (1955) and its concepts provide many useful mathematical tools to deal with many real-life problems. Topological structures (such as "Near concepts") have played an important role in expansion of some theories, devoted for discussing uncertainty, such as rough sets theory, fuzzy sets theory and probability theory. In this section notion of near open sets is being introduced in case of soft rough sets. Their properties are illustrated and the relationships among them and T_{SR}-approximations (El-Bably et al. 2021) are superimposed. Relationships among these different approximations are elaborated with the help of some examples. Notion of accuracy measures helps us to have a comparison among different types of approximations available in literature. In the present study, a such comparison is given for the proposed method and already studied in Feng et al. (2011) and El-Bably et al. (2021).

Definition 3.1 Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, by using the concept of near open sets we can define the following sets:

(i) T_{SR}-Preopen (briefly P_{SR}-open) set, if $X \subseteq \underline{T_{SR}}(\overline{T}_{SR}(X))$.
(ii) T_{SR}-Semi open (briefly S_{SR}-open) set, if $X \subseteq \overline{T}_{SR}(\underline{T_{SR}}(X))$.
(iii) γ_{SR}-open set, if $X \subseteq [\underline{T_{SR}}(\overline{T}_{SR}(X)) \cup \overline{T}_{SR}(\underline{T_{SR}}(X))]$.
(iv) α_{SR}-open set, if $X \subseteq \underline{T_{SR}}(\overline{T}_{SR}(\underline{T_{SR}}(X)))$.
(v) β_{SR}-open set, if $X \subseteq \overline{T}_{SR}(\underline{T_{SR}}(\overline{T}_{SR}(X)))$.

Remark 3.1 (i) The above sets are called "J_{SR}-near open" sets and the families of J_{SR}-near open sets of U denoted by $J_{SR}O(U)$, for each $J \in \{P, S, \gamma, \alpha, \beta\}$.

(ii) The complement of J_{SR}-near open set is called "J_{SR}-near closed" set and the families of J_{SR}-near closed sets of U denoted by $J_{SR}C(U)$, for each $J \in \{P, S, \gamma, \alpha, \beta\}$.

(iii) According to the results of near open sets (Abd El-Monsef et al. 1983; Andrijevi 1996, 1986; El-Bably 2015; Levine 1963; Mashhour et al. 1982; Njestad 1965), the implications between T_{SR}-sets and T_{SR}-near sets are given in Fig. 1.

Remark 3.2 The above relationships can be showing in Examples 3.1 and 3.2. In addition, these examples confirm that the reverse implications, of the above relationships, are not true in general. Moreover, it illustrates that the different types of two classes $J_{SR}O(U)$ and $J_{SR}C(U)$ are not comparable.

Example 3.1 Let F_A be a soft set over U and $A_{SR} = (U, F_A)$ a soft approximation space, where $U = \{u_1, u_2, u_3, u_4\}$, and $A = \{e_1, e_2, e_3, e_4\}$ such that $F_A = \{(e_1, \{u_4\}), (e_2, \{u_1, u_2, u_4\}), (e_3, \{u_1, u_2\}), (e_4, U)\}$. Then, we get:

The soft subbasis of T_{SR} is: $S_{F_A} = \{\{u_4\}, \{u_1, u_2\}, \{u_1, u_2, u_4\}, U\}$.
The soft basis of T_{SR} is: $\mathbf{B}_{F_A} = \{\phi, \{u_4\}, \{u_1, u_2\}, \{u_1, u_2, u_4\}, U\}$.
Soft rough topology is: $T_{SR} = \{U, \phi, \{u_4\}, \{u_1, u_2\}, \{u_1, u_2, u_4\}\}$.
The complement of T_{SR} is: ${T_{SR}}^c = \{U, \phi, \{u_3\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}\}$.

Accordingly, the classes of J_{SR}-near open sets of U are:

$P_{SR}O(U) = \{U, \phi, \{u_1\}, \{u_2\}, \{u_4\}, \{u_1, u_2\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}$.
$S_{SR}O(U) = \{U, \phi, \{u_4\}, \{u_1, u_2\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}\}$.
$\gamma_{SR}O(U) = \{U, \phi, \{u_1\}, \{u_2\}, \{u_4\}, \{u_1, u_2\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}$.
$\alpha_{SR}O(U) = \{U, \phi, \{u_4\}, \{u_1, u_2\}, \{u_1, u_2, u_4\}\}$.
$\beta_{SR}O(U) = \{U, \phi, \{u_1\}, \{u_2\}, \{u_4\}, \{u_1, u_2\}, \{u_1, u_3\}, \{u_1, u_4\}, \{u_2, u_3\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}$.

By taking the complements of the above classes, we get the classes of all J_{SR}-closed sets $J_{SR}C(U)$.

Remark 3.3 In the above example, the topology T_{SR} (resp. ${T_{SR}}^c$) and the class of α_{SR}-open sets $\alpha_{SR}O(U)$ (resp. α_{SR}-closed sets $\alpha_{SR}C(U)$) are equal. The following example illustrates the relationship between the topology T_{SR} (resp. ${T_{SR}}^c$) and the class of α_{SR}-open sets $\alpha_{SR}O(U)$ (resp. α_{SR}-closed sets $\alpha_{SR}C(U)$).

Example 3.2 Let F_A be a soft set over U and $A_{SR} = (U, F_A)$ a soft approximation space, where $U = \{a, b, c\}$, and $A = \{e_1, e_2, e_3, e_4\}$ such that $F_A = \{(e_1, \{a\}), (e_2, \{b\}), (e_3, \{a, b\}), (e_4, U)\}$. Then, we get:

Soft rough topology is: $T_{SR} = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$.
The complement of T_{SR} is: ${T_{SR}}^c = \{U, \phi, \{c\}, \{a, c\}, \{b, c\}\}$.
Accordingly, the class of α_{SR}-open sets of U is: $\alpha_{SR}O(U) = \{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$.
The class of α_{SR}-closed sets of U is: $\alpha_{SR}C(U) = \{U, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.
Obviously, $\alpha_{SR}O(U) \neq T_{SR}$ and $\alpha_{SR}C(U) \neq {T_{SR}}^c$.

By using the above classes "J_{SR}-near open" sets, we introduce new methods for approximating soft rough sets as the following definitions illustrate.

Definition 3.2 Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$ we define the J_{SR}-near soft rough approximations ("J_{SR}-lower," and J_{SR}-upper), respectively, by:

$$\underline{J_{SR}}(X) = \cup\{G \in J_{SR}O(U) : G \subseteq X\},$$
$$\overline{J_{SR}}(X) = \cap\{H \in J_{SR}C(U) : X \subseteq H\}.$$

Definition 3.3 Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$ we define the "J_{SR}-positive, J_{SR}-negative, J_{SR}-boundary" regions and the "J_{SR}-accuracy" of the J_{SR}- near approximations, respectively, by:

$$POS_{J_{SR}}(X) = \underline{J_{SR}}(X),$$
$$NEG_{J_{SR}}(X) = U - \overline{J_{SR}}(X),$$
$$BND_{J_{SR}}(X) = \overline{J_{SR}}(X) - \underline{J_{SR}}(X) \quad \text{and}$$
$$\mu_{J_{SR}}(X) = \begin{cases} \frac{|\underline{J_{SR}}(X)|}{|\overline{J_{SR}}(X)|} & \text{if } X \neq \phi, \\ 1 & \text{otherwise.} \end{cases}$$

Remark 3.4 (i) It is clear that $0 \leq \mu_{J_{SR}}(X) \leq 1$, for any $X \subseteq U$.

(ii) If $\overline{J_{SR}}(X) = \underline{J_{SR}}(X)$, then $BND_{J_{SR}}(X) = \phi$ and $\mu_{J_{SR}}(X) = 1$. Thus, $X \subseteq U$ is said to be "J_{SR}-definable" or "J_{SR}-exact" set; otherwise, X is called a "J_{SR}-rough" set.

Remark 3.5 In any a T_{SR}-approximation space, we can compute the J_{SR}-near approximations of any subset directly, without computing the classes of J_{SR}-near sets, by using the T_{SR}-lower and T_{SR}-upper approximations as the following theorem illustrates.

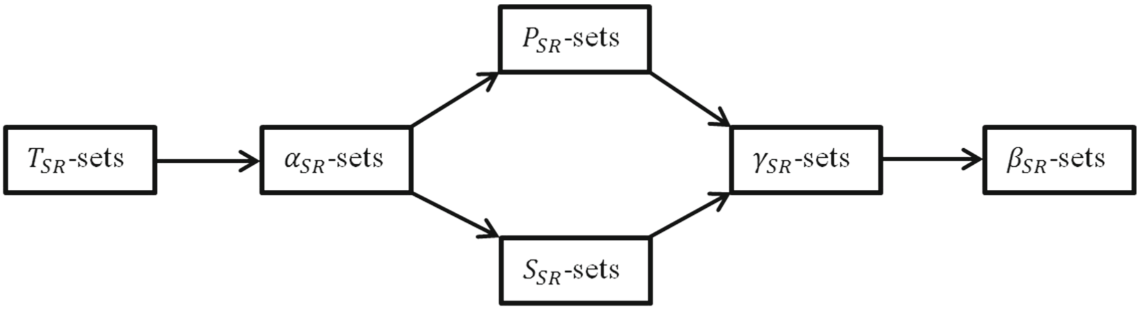

Fig. 1 Relationships

Theorem 3.1 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$:*

(i) $\underline{P_{SR}}(X) = X \cap \underline{T_{SR}}(\overline{T_{SR}}(X))$.
(ii) $\underline{S_{SR}}(X) = X \cap \overline{T_{SR}}(\underline{T_{SR}}(X))$.
(iii) $\underline{\gamma_{SR}}(X) = \underline{P_{SR}}(X) \cup \underline{S_{SR}}(X)$.
(iv) $\underline{\alpha_{SR}}(X) = X \cap \underline{T_{SR}}(\overline{T_{SR}}(\underline{T_{SR}}(X)))$.
(v) $\underline{\beta_{SR}}(X) = X \cap \overline{T_{SR}}(\underline{T_{SR}}(\overline{T_{SR}}(X)))$.
(vi) $\overline{P_{SR}}(X) = X \cup \overline{T_{SR}}(\underline{T_{SR}}(X))$.
(vii) $\overline{S_{SR}}(X) = X \cup \underline{T_{SR}}(\overline{T_{SR}}(X))$.
(viii) $\overline{\gamma_{SR}}(X) = \overline{P_{SR}}(X) \cap \overline{S_{SR}}(X)$.
(ix) $\overline{\alpha_{SR}}(X) = X \cup \overline{T_{SR}}(\underline{T_{SR}}(\overline{T_{SR}}(X)))$.
(x) $\overline{\beta_{SR}}(X) = X \cup \underline{T_{SR}}(\overline{T_{SR}}(\underline{T_{SR}}(X)))$.

Proof Here, proof of only first statement is given; remaining can be shown similarly.

(i) Firstly, $\underline{P_{SR}}(X) \subseteq X$. since $\underline{P_{SR}}(X)$ is P_{SR}-open set, then $\underline{P_{SR}}(X) \subseteq \underline{T_{SR}}(\overline{T_{SR}}(\underline{P_{SR}}(X))) \subseteq \underline{T_{SR}}(\overline{T_{SR}}(X))$. Thus, $\underline{P_{SR}}(X) \subseteq X \cap \underline{T_{SR}}(\overline{T_{SR}}(X))$. Conversely, since $X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \subseteq \underline{T_{SR}}(\overline{T_{SR}}(X))$ and $\underline{T_{SR}}(\overline{T_{SR}}(X)) = \underline{T_{SR}}(\overline{T_{SR}}(X)) \cap \underline{T_{SR}}(\overline{T_{SR}}(\underline{T_{SR}}(\overline{T_{SR}}(X))))$. Then, $X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \subseteq \underline{T_{SR}}(\overline{T_{SR}}(X)) \cap \underline{T_{SR}}(\overline{T_{SR}}(\underline{T_{SR}}(\overline{T_{SR}}(X))))$. Accordingly, $X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \subseteq \underline{T_{SR}}(\overline{T_{SR}}(X \cap \underline{T_{SR}}(\overline{T_{SR}}(X))))$, which means that $X \cap \underline{T_{SR}}(\overline{T_{SR}}(X))$ is P_{SR}-open set contained in X. Hence, $X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \subseteq \underline{P_{SR}}(X)$. □

The following proposition gives the fundamental properties of J_{SR}-near approximations.

Proposition 3.1 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X, Y \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$:*

(i) $\underline{J_{SR}}(X) \subseteq X \subseteq \overline{J_{SR}}(X)$.
(ii) $\underline{J_{SR}}(U) = \overline{J_{SR}}(U) = U$.
(iii) $\underline{J_{SR}}(\phi) = \overline{J_{SR}}(\phi) = \phi$.
(iv) *If $X \subseteq Y$, then $\underline{J_{SR}}(X) \subseteq \underline{J_{SR}}(Y)$.*
(v) *If $X \subseteq Y$, then $\overline{J_{SR}}(X) \subseteq \overline{J_{SR}}(Y)$.*
(vi) $\underline{J_{SR}}(X \cap Y) \subseteq \underline{J_{SR}}(X) \cap \underline{J_{SR}}(Y)$.
(vii) $\overline{J_{SR}}(X \cap Y) \subseteq \overline{J_{SR}}(X) \cap \overline{J_{SR}}(Y)$.
(viii) $\underline{J_{SR}}(X \cup Y) \supseteq \underline{J_{SR}}(X) \cup \underline{J_{SR}}(Y)$.
(ix) $\overline{J_{SR}}(X \cup Y) \supseteq \overline{J_{SR}}(X) \cup \overline{J_{SR}}(Y)$.
(x) $\underline{J_{SR}}(X) = (\overline{J_{SR}}(X^c))^c$.
(xi) $\overline{J_{SR}}(X) = (\underline{J_{SR}}(X^c))^c$.
(xii) $\underline{J_{SR}}(\underline{J_{SR}}(X)) = \underline{J_{SR}}(X)$.
(xiii) $\overline{J_{SR}}(\overline{J_{SR}}(X)) = \overline{J_{SR}}(X)$.

Proof The statements (i), (ii) and (iii) directly from Definition 3.2. (iv) Let $X \subseteq Y$, then $\underline{J_{SR}}(X) = X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \subseteq Y \cap \underline{T_{SR}}(\overline{T_{SR}}(Y)) = \underline{J_{SR}}(Y)$. (v) By similar way as (iv). (vi) By using (iv), we get $\underline{J_{SR}}(X \cap Y) = (X \cap Y) \cap \underline{T_{SR}}(\overline{T_{SR}}(X \cap Y))$. But $X \cap Y \subseteq X$ and $X \cap Y \subseteq Y$, this implies $\overline{T_{SR}}(X \cap Y) \subseteq \overline{T_{SR}}(X)$ and $\overline{T_{SR}}(X \cap Y) \subseteq \overline{T_{SR}}(Y)$. Thus, $(X \cap Y) \cap \underline{T_{SR}}(\overline{T_{SR}}(X \cap Y)) \subseteq (X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) \cap (Y \cap \underline{T_{SR}}(\overline{T_{SR}}(Y))$. Accordingly, $\underline{J_{SR}}(X \cap Y) \subseteq \underline{J_{SR}}(X) \cap \underline{J_{SR}}(Y)$. (vii), (viii), and (ix) By same way as (vi). (x) $(\overline{J_{SR}}(X^c))^c = (X^c \cup \overline{T_{SR}}(\underline{T_{SR}}(X^c)))^c = X \cap \underline{T_{SR}}(\overline{T_{SR}}(X)) = \underline{J_{SR}}(X)$. (xi) By similar way as (x). (xii) Since $\underline{J_{SR}}(X)$ is J_{SR}-near open set, then $\underline{J_{SR}}(\underline{J_{SR}}(X)) = \underline{J_{SR}}(X)$. (xiii) By similar way as (xii). □

The main goal of the following results is to show the relationships among soft rough approximations (Feng et al. 2011), T_{SR}-approximations (El-Bably et al. 2021) and J_{SR}-near approximations. Moreover, these results illustrate the importance of using J_{SR}-near concepts in soft rough set context.

Theorem 3.2 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$:*

(i) $\underline{Apr_{SR}}(X) \subseteq \underline{T_{SR}}(X) \subseteq \underline{J_{SR}}(X)$.
(ii) $\overline{J_{SR}}(X) \subseteq \overline{T_{SR}}(X) \subseteq \overline{Apr_{SR}}(X)$.

Proof We will prove the first statement and the second is similar.

Table 1 Comparison between the T_{SR}-accuracy and J_{SR}-near accuracy "current methods"

$P(U)$	$\mu_{T_{SR}}(X)$ (El-Bably et al. 2021)	$\mu_{\alpha_{SR}}(X)$	$\mu_{P_{SR}}(X)$	$\mu_{S_{SR}}(X)$	$\mu_{\gamma_{SR}}(X)$	$\mu_{\beta_{SR}}(X)$
$\{u_1\}$	0	0	1	0	1	1
$\{u_2\}$	0	0	1	0	1	1
$\{u_3\}$	0	0	0	0	0	0
$\{u_4\}$	1/2	1/2	1/2	1	1	1
$\{u_1, u_2\}$	2/3	2/3	2/3	1	1	1
$\{u_1, u_3\}$	0	0	1/2	0	1/2	1
$\{u_1, u_4\}$	1/4	1/4	2/3	1/4	2/3	1
$\{u_2, u_3\}$	0	0	1/2	0	1/2	1
$\{u_2, u_4\}$	1/4	1/4	2/3	1/4	2/3	1
$\{u_3, u_4\}$	1/2	1/2	1/2	1	1	1
$\{u_1, u_2, u_3\}$	2/3	2/3	2/3	1	1	1
$\{u_1, u_2, u_4\}$	3/4	3/4	3/4	3/4	3/4	3/4
$\{u_1, u_3, u_4\}$	1/4	1/4	1	1/2	1	1
$\{u_2, u_3, u_4\}$	1/4	1/4	1	1/2	1	1
U	1	1	1	1	1	1
ϕ	1	1	1	1	1	1

(i) Firstly, by Theorem 2.3.1 (El-Bably et al. 2021), $\underline{Apr_{SR}}(X) \subseteq \underline{T_{SR}}(X)$. Now, since the families of J_{SR}-near open sets $J_{SR}O(U)$ are larger than the topologies T_{SR}. Therefore, we get $\underline{T_{SR}}(X) = \cup\{G \in T_{SR} : G \subseteq X\} \subseteq \cup\{G \in J_{SR}O(U) : G \subseteq X\} = \underline{J_{SR}}(X)$. □

Corollary 3.1 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$:*

(i) $BND_{J_{SR}}(X) \subseteq BND_{T_{SR}}(X) \subseteq BND_{A_{SR}}(X)$.
(ii) $\mu_{J_{SR}}(X) \geq \mu_{T_{SR}}(X) \geq \mu_{A_{SR}}(X)$.

Theorem 3.3 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$: X is a A_{SR}-exact set $\Rightarrow$ X is a T_{SR}-exact set $\Rightarrow$ X is a J_{SR}-near exact set.*

Proof First, Corollary 2.3.2 (El-Bably et al. 2021), X is a A_{SR}-exact set $\Rightarrow$ X is a T_{SR}-exact set. Now, let X is a T_{SR}-exact $\Rightarrow BND_{T_{SR}}(X) = \phi$. By using Corollary 3.1, $\Rightarrow BND_{J_{SR}}(X) = \phi$, and hence, X is a J_{SR}-near exact set. □

Remark 3.6 The converse of the above results is not true in general as illustrated in Example 3.3.

Example 3.3 Consider Example 3.1, we compute T_{SR}-approximations and J_{SR}-near approximations of all subsets in U, and then, we get comparisons among T_{SR}-accuracy and J_{SR}-near accuracy of the approximations as illustrated in Table 1.

Remark 3.7 From Table 1, we notice that:

(i) There are different methods to approximate the subsets of a set. The best approximations are obtained by employing β_{SR}, because here the boundary regions are reduced or vanished. When β_{SR} is considered as an approximation space, here size of lower approximations is larger and size of upper approximations is smaller as compared to other proposed approximation spaces. Therefore, suggested techniques of this paper play an important role in removing the vagueness (uncertainty) of rough sets. For example, some of the subsets in the above table are exact in β_{SR}, but rough in the other approximation spaces.
(ii) In addition, the values of accuracy measures in β_{SR}-approximation space are higher than other approximation spaces, since for any subset $X \subseteq U$ we have $\mu_{\beta_{SR}}(X) \geq \mu_{\gamma_{SR}}(X) \geq \mu_{P_{SR}}(X) \geq \mu_{\alpha_{SR}}(X) \geq \mu_{T_{SR}}(X)$ and $\mu_{\beta_{SR}}(X) \geq \mu_{\gamma_{SR}}(X) \geq \mu_{S_{SR}}(X) \geq \mu_{\alpha_{SR}}(X) \geq \mu_{T_{SR}}(X)$. Thus, it can be said that J_{SR}-near soft rough approximation spaces help us to extract and discover the hidden knowledge in data which is collected in real-life situations.
(iii) The accuracy measure of T_{SR}-approximation space is equal to the accuracy measure of α_{SR}-approximation space, since the topology T_{SR} and $\alpha_{SR}O(U)$ are the same in the present example (Example 3.1). But if we consider Example 3.2, we get $\mu_{\alpha_{SR}}(X) \geq \mu_{T_{SR}}(X)$, for each $X \subseteq U$. For example, the subsets $X = \{b\}$ and $Y = \{a, c\}$ are exact in $\alpha_{SR}O(U)$ approximation space (since $\underline{\alpha_{SR}}(X) = \overline{\alpha_{SR}}(X) = X$ and $\underline{\alpha_{SR}}(Y) = \overline{\alpha_{SR}}(Y) = Y$). But in T_{SR}-approximation space these are rough. Because $\underline{T_{SR}}(X) = \{b\}$, $\overline{T_{SR}}(X) = \{b, c\}$

Table 2 Tabular representation for soft set F_A in diabetes mellitus

Patients	p_1	p_2	p_3	p_4	p_5	p_6
e_1	1	0	0	1	1	1
e_2	1	1	1	0	0	0
e_3	0	1	0	1	1	1
Diabetes	0	1	1	1	1	1

and $\underline{T_{SR}}(Y) = \{a\}, \overline{T_{SR}}(Y) = \{a, c\}$). Moreover, $\mu_{\alpha_{SR}}(X) = 1$ and $\mu_{\alpha_{SR}}(Y) = 1$. But $\mu_{T_{SR}}(X) = 1/2$ and $\mu_{T_{SR}}(Y) = 1/2$.

The relationships among different types of the J_{SR}-near approximations, J_{SR}-near boundary and J_{SR}-near accuracy are given by the following results.

Proposition 3.2 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, the following statements are true in general.*

(i) $\underline{\alpha_{SR}}(X) \subseteq \underline{P_{SR}}(X) \subseteq \underline{\gamma_{SR}}(X) \subseteq \underline{\beta_{SR}}(X)$.
(ii) $\underline{\alpha_{SR}}(X) \subseteq \underline{S_{SR}}(X) \subseteq \underline{\gamma_{SR}}(X) \subseteq \underline{\beta_{SR}}(X)$.
(iii) $\overline{\beta_{SR}}(X) \subseteq \overline{\gamma}_{SR}(X) \subseteq \overline{P_{SR}}(X) \subseteq \overline{\alpha_{SR}}(X)$.
(iv) $\overline{\beta_{SR}}(X) \subseteq \overline{\gamma_{SR}}(X) \subseteq \overline{S_{SR}}(X) \subseteq \overline{\alpha_{SR}}(X)$.

Proof By using the implications between the different families of J_{SR}-near open sets, the proof is obvious. □

Corollary 3.2 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, the following statements are true in general.*

(i) $BND_{\beta_{SR}}(X) \subseteq BND_{\gamma_{SR}}(X) \subseteq BND_{P_{SR}}(X) \subseteq BND_{\alpha_{SR}}(X)$.
(ii) $BND_{\beta_{SR}}(X) \subseteq BND_{\gamma_{SR}}(X) \subseteq BND_{S_{SR}}(X) \subseteq BND_{\alpha_{SR}}(X)$.
(iii) $\mu_{\beta_{SR}}(X) \geq \mu_{\gamma_{SR}}(X) \geq \mu_{P_{SR}}(X) \geq \mu_{\alpha_{SR}}(X)$.
(iv) $\mu_{\beta_{SR}}(X) \geq \mu_{\gamma_{SR}}(X) \geq \mu_{S_{SR}}(X) \geq \mu_{\alpha_{SR}}(X)$.

Corollary 3.3 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X \subseteq U$. Then, the following statements are true in general.*

(i) *X is a α_{SR}-exact $\Rightarrow$ X is a P_{SR}-exact X is a γ_{SR}-exact $\Rightarrow$ X is a β_{SR}-exact.*
(ii) *X is a α_{SR}-exact $\Rightarrow$ X is a S_{SR}-exact X is a γ_{SR}-exact $\Rightarrow$ X is a β_{SR}-exact.*

Remark 3.7 The converse of the above results is not true in general as Example 3.2 illustrated.

4 Degree of accuracy for generalized soft rough sets

In the present section, we study the degree of accuracy for generalized soft rough sets using different types of accuracy measures that given in El-Bably et al. (2021) and the current paper. Moreover, we study some of its properties in more depth and define equivalence relations generated from the accuracy measures. According to Definition 2.3.5 (El-Bably et al. (2021)), using the T_{SR}-accuracy of T_{SR}-approximations (resp. Definition 3.3, using the J_{SR}-accuracy of J_{SR}-approximations), we may define a relation on $P(U)$ as the following definition illustrated.

Definition 4.1 Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X, Y \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$, it said that $X \sim_k Y$ if and only if $\mu_k(X) = \mu_k(Y)$, such that $k \in \{T_{SR}, J_{SR}\}$.

Theorem 4.1 *Let $A_{T_{SR}} = (U, F_A, T_{SR})$ be a T_{SR}-approximation space and $X, Y \subseteq U$. Then, for each $J \in \{P, S, \gamma, \alpha, \beta\}$ the relation $\sim_k$ is an equivalence relation on $P(U)$ and the partition $P(U)/\sim_k$ maintains a strict order among its elements, such that $k \in \{T_{SR}, J_{SR}\}$.*

Proof It is easy to see that the relation $\sim_k$ defined on $P(U)$ is an equivalence relation. Therefore, $P(U)/\sim_k$ is a partition on $P(U)$. If a class in $P(U)/\sim_k$ containing an element $X \in P(U)$ is denoted by $[X]_{\sim_k}$, then by Definition 4.1, for each $Y \in [X]_{\sim_k}$ we have $\mu_k(X) = \mu_k(Y)$. This means that each element of $P(U)/\sim_k$ can be characterized by a unique real number from [0, 1]. For the class $[X]_{\sim_k}$, let this number be represented by a and we call it characteristic of $[X]_{\sim_k}$. Since each class in $P(U)/\sim_k$ has a unique characteristic belonging to [0, 1], there is a strict order among the classes. Hence, we define this order as:

$$[X]_{\sim_k} \prec [Y]_{\sim_k} \quad \text{if and only if} \quad a < b,$$

where b is characteristic of the class$[Y]_{\sim_k} \in P(U)/\sim_k$. □

Remark 4.1 According to Theorem 4.1, there are six different equivalence relations can be defined on $P(U)$. Therefore, these relations classify $P(U)$ into distinct classes which have a strict order among them. Example 4.1 illustrates this fact.

Example 4.1 According to Example 3.2, and by using Definition 4.1, we can get the following equivalence classes:

– The equivalence classes of $\sim_{T_{SR}}$ are: $P(U)/\sim_{T_{SR}} = \{\{\{u_1\}, \{u_2\}, \{u_3\}, \{u_2, u_3\}\}, \{\{u_4\}, \{u_3, u_4\}\}, \{\{u_1, u_2\}, \{u_1, u_2, u_3\}\}, \{\{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}, \{\{u_1, u_2, u_4\}\}, \{U, \phi\}\}$.

Table 3 Comparisons among the soft rough approximations Feng et al. (2011), T_{SR}-approximations El-Bably et al. (2021) and β_{SR}-approximations (given in current paper in Definition 3.2)

X	$\{p_1, p_2\}$	$\{p_1, p_3\}$	$\{p_2, p_5, p_6\}$	$\{p_2, p_3, p_4, p_5, p_6\}$
$\underline{Apr_{SR}}(X)$	ϕ	ϕ	ϕ	$\{p_2, p_4, p_5, p_6\}$
$\overline{Apr_{SR}}(X)$	U	U	$\{p_1, p_2, p_4, p_5, p_6\}$	U
$BND_{SR}(X)$	U	U	$\{p_1, p_2, p_4, p_5, p_6\}$	$\{p_3\}$
$\mu_{SR}(X)$	0	0	0	2/3
$\underline{T_{SR}}(X)$	$\{p_1\}$	$\{p_1\}$	$\{p_4, p_5, p_6\}$	$\{p_2, p_4, p_5, p_6\}$
$\overline{T_{SR}}(X)$	$\{p_1, p_2, p_3\}$	$\{p_1, p_3\}$	$\{p_4, p_5, p_6\}$	$\{p_2, p_3, p_4, p_5, p_6\}$
$BND_{T_{SR}}(X)$	$\{p_2, p_3\}$	$\{p_3\}$	ϕ	$\{p_3\}$
$\mu_{T_{SR}}(X)$	1/3	1/2	1	2/3
$\underline{\beta_{SR}}(X)$	$\{p_1, p_2\}$	$\{p_1, p_3\}$	$\{p_4, p_5, p_6\}$	$\{p_2, p_3, p_4, p_5, p_6\}$
$\overline{\beta_{SR}}(X)$	$\{p_1, p_2\}$	$\{p_1, p_3\}$	$\{p_4, p_5, p_6\}$	$\{p_2, p_3, p_4, p_5, p_6\}$
$BND_{\beta_{SR}}(X)$	ϕ	ϕ	ϕ	ϕ
$\mu_{\beta_{SR}}(X)$	1	1	1	1

Table 4 An algorithm to decision making using βSR-approximations

Algorithm 4.1	A decision making via β_{SR}-approximations
Step 1:	Input the soft set (F, A)
Step 2:	Take the class $S_{F_A} = \{F(e) : \forall e \in A\}$ as a subbasis for a basis $\mathbf{B}_{F_A}$
Step 3:	Compute the basis $\mathbf{B}_{F_A} = \{X \cap Y : (X, Y) \in S_{F_A} \times S_{F_A}\}$ by Definition 2.3.1
Step 4:	Generate the topology $T_{SR} = \cup\{B : B \in \mathbf{B}_{F_A}\}$ by Definition 2.3.2
Step 5:	Using Definition 3.2 to compute the class of β_{SR}-near sets to investigate the $\underline{\beta_{SR}}(X)$ and $\overline{\beta_{SR}}(X)$, for every $X \subseteq U$
Step 6:	Determine the boundary region $BND_{\beta_{SR}}(X)$ from Step 5. According to Definition 3.3
Step 7:	Calculate the accuracy of the approximation $\mu_{\beta_{SR}}(X)$ from Step 5. According to Definition 3.3
Step 8:	Decide, exactly, rough sets and exact sets. Using Definition 3.3

Order among the above classes is given by: $\{U, \phi\} > \{\{u_1, u_2, u_4\}\} > \{\{u_1, u_2\}, \{u_1, u_2, u_3\}\} > \{\{u_4\}, \{u_3, u_4\}\} > \{\{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\} > \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_2, u_3\}\}$.

Further, the equivalence classes obtained by the relation $\sim_{P_{SR}}$ are: $P(U)/\sim_{P_{SR}} = \{\{U, \phi, \{u_1\}, \{u_2\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}, \{\{u_4\}, \{u_1, u_3\}, \{u_2, u_3\}, \{u_3, u_4\}\}, \{\{u_3\}\}, \{\{u_1, u_2\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_2, u_3\}\}, \{\{u_1, u_2, u_4\}\}\}$.

Order among the above classes is given by: $\{U, \phi, \{u_1\}, \{u_2\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\} > \{\{u_1, u_2, u_4\}\} > \{\{u_1, u_2\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_2, u_3\}\} > \{\{u_4\}, \{u_1, u_3\}, \{u_2, u_3\}, \{u_3, u_4\}\} > \{\{u_3\}\}$.

Similarly for the remaining equivalence relations their classes, and their orders can be determined easily.

5 Application of T_{SR} Approximation Space in decision-making problems

In this section, we will apply the concept of soft rough topology T_{SR} in diabetes mellitus (DM) (Bakeir et al. 2018), commonly referred to as diabetes, and is a group of metabolic diseases in which there are high blood sugar levels over a prolonged period. Symptoms of high blood sugar include frequent urination, increased thirst and increased hunger. If left untreated, diabetes can cause many complications. Acute complications can include diabetic ketoacidosis, non-ketotic hyperosmolar coma or death. Serious long-term complications include heart disease, stroke, chronic kidney failure, foot ulcers and damage to the eyes. Consider the following information table giving data about 6 patients as a random representative. The rows of the table represent the attributes (the symptoms for Diabetes), and the columns represent the objects (the patients). Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ and $A = \{e_1, e_2, e_3\}$, where $e_1 = IncreasedHunger$, $e_2 = FrequentUrination$ and $e_3 = IncreasedThirst$. Let $A_S = (U, F_A)$, where F_A is a soft set over U given in Table 2.

Then, we get: $T_{SR} = \{U, \phi, \{p_1\}, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}, \{p_1, p_4, p_5, p_6\}, \{p_2, p_4, p_5, p_6\}, \{p_1, p_2, p_4, p_5, p_6\}\}$ and $T_{SR}{}^c = \{U, \phi, \{p_3\}, \{p_1, p_3\}, \{p_2, p_3\}, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}, \{p_2, p_3, p_4, p_5, p_6\}\}$. Now, we find the approximations of some subsets using A_{SR}-approximation space proposed in Feng et al. (2011), T_{SR}-approximation space (El-Bably et al. 2021) and β_{SR}-approximation space as in Table 3.

Remark 5.1 From Table 3, we can notice the following:

(1) The set of patients having diabetes is $X = \{p_2, p_3, p_4, p_5, p_6\}$. Thus, from Table 3, we have X is a β_{SR}-definable set. But in A_{SR}-approximation space (Feng et al. 2011) and T_{SR}-approximation space (El-Bably et al. 2021), X is a rough set. This implies that no patient is suffering from diabetes which contradicts Table 4. This means that in A_{SR}-approximation space and T_{SR}-approximation space, we are unable to decide for any element of U whether it belongs to X or X_c.

(2) It can be seen that β_{SR}-approximation space provides more accurate approximations for certain subsets as compared with already existing approximation spaces.

6 Conclusion

In this article, some new approximation spaces called topological soft rough approximation spaces have been introduced. In proposed approximation spaces, topology generated from a soft set plays a vital role. Properties of approximations, of any subset of a set, in these approximation spaces have been studied here. Their relationships with soft rough approximations have been examined. In fact, we have investigated that in proposed approximation spaces all properties similar to that of Pawlak's rough sets may be satisfied without imposing any extra condition. Comparison among proposed and previous works in Feng et al. (2011) and El-Bably et al. (2021) has been provided. Thus, it can be said that proposed method is more suitable than as given in Feng et al. (2011) for decision-making problems. Thus, these methods are very useful in real-life applications.

Finally, we have introduced an application of proposed methods in decision making for diagnosis, namely in diabetes mellitus (Bakeir et al. 2018), to illustrate the importance of current methods. It provides a comparison between proposed methods with already existing in the literature. An algorithm is given for the application of given method. More importantly, the present paper not only provides a complete new range of approximation spaces but also increases the accuracy of approximations of the subsets of a set.

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Declaration

Conflict of interest Authors do not have any conflict of interest with any other person or organization.

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