Analysis of $\alpha$-nucleus elastic and inelastic scattering using the single folding $\alpha$-cluster model

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Analysis of $\pi^\pm$-nucleus elastic and inelastic scattering using the single folding $\alpha$-cluster model

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Abstract
An analysis of $\pi^\pm$-nucleus elastic and inelastic scattering has been performed using local semi-microscopic optical potentials constructed in the framework of the single folding approach. The folding calculations are carried out based upon the $\alpha$-cluster structure of the target nuclei with two different phenomenological forms of the pion–alpha effective interaction. The derived potentials have been employed to extract the angular distributions of elastic and inelastic scattering cross sections through a broad energy range, 100–766 MeV, where 39 sets of data have been successfully described. The corresponding reaction and total cross sections have also been extracted.

Keywords: cluster model, nuclear structure, distorted wave models, collective models

1. Introduction

The descriptions of pion scattering reactions are more challenging than those for other hadronic probes. Thus, we must ensure that our reaction models are secure, even with a range of nuclear structure models. Only then can we be certain that any conclusion we draw from pion data and analysis is secure and universal. A pion analysis is challenging due to the magnitudes of the cross sections which are very sensitive to beam energies, a strong p-wave resonance that is not seen for nucleons and other light projectiles, a sensitivity to both neutron and proton transitions and their interference, and true absorption, a strong reaction channel not met with protons etc. We should therefore continue to test our understandings with respect to existing data with studies of the effects of both reaction models and nuclear structure models. In this paper, the nuclear model is based on a folding of the interaction from pions on the cluster components of the carbon or oxygen targets in a local equivalent version of the nonlocal interaction. This local model is much easier to use with existing reaction models and codes than the nonlocal equivalents, and so this work tests two features of pion-nucleus reactions.

At around 200 MeV, strong absorption caused by the resonance character of the pion-nucleon amplitude is dominant in the pion–nucleus scattering. This scattering is insensitive to the finer details of the pion-nucleus interaction. In the limit of s and p waves only, different studies of differential, reaction, and total cross sections have used the impulse approximation to construct an optical potential for pions. These studies show that the optical potential results in a reasonable agreement with the experimental data when used in a relativistic optical model code or in a semiclassical calculation [1].

The pion-nucleus scattering, particularly in the $\Delta$-resonance region, can be analyzed using the Klein–Gordan equation (KGE) based upon a nonlocal optical potential [2]. However, it was interesting to consider another treatment by localizing the nonlocal potential in order to obtain a different point of view of the dynamics of pion scattering. In this context, Satchler [3] reduced the KGE into the form of a Schrödinger equation (SE) by introducing some redefined kinematical quantities. He [3] introduced a local phenomenological optical potential of a Woods–Saxon (WS) shape to describe—with great success—the angular distributions of the differential cross sections for elastic and inelastic scattering of positive and negative pions ($\pi^\pm$) from the nuclei $^{40,48}$Ca, $^{58}$Ni, $^{90}$Zr, $^{118}$Sn, and $^{208}$Pb at energies ranging from 100 to 300 MeV. The success of this treatment motivated several analyses, over the last two decades, of pion-nucleus scattering over a wide range of pion beam energies by introducing a
simple phenomenological WS form of the local optical potentials into the nonrelativistic SE, see e.g. [4–8].

However, the $\alpha$-cluster model has been successfully employed in order to describe light heavy ions (HI) reactions [9–19] in the framework of single and double folding approaches to extract semi-microscopic representation of the $\alpha$-nucleus and nucleus–nucleus potentials, respectively. These potentials have proved to give a good description of light HI elastic scattering data. The results of these studies highlighted the semi-microscopic description as a successful alternative to the macroscopic one constructed by folding an appropriate effective nucleon–nucleon (NN) interaction over the matter density distributions of the interacting nuclei [20–24]. In this context, in two very recent studies [25, 26], $\pi^\pm$ and $K^\pm$ elastic and inelastic scattering from $^3\text{Li}$ and $^{12}\text{C}$ have been analyzed by applying the Watanabe superposition model with a phenomenological WS shape. Satisfactory predictions of the data have been obtained. So, following this merit, we find it is interesting to extend the application of the $\alpha$-cluster formalism to derive a folded form of the pion–nucleus local optical potential. It is worth mentioning that, due to the lack of recent measured scattering data, carrying out research to extract microscopic or semi-microscopic potentials is recommended in order to obtain successful predictions of unavailable data.

The present study is devoted to extracting semi-microscopic optical potentials in the framework of the single folding (SF) approach in order to analyze the Watanabe potential to be used in the SF expression (1). The first one is assumed to be in a Gaussian form as

$$V_{\pi-a}^{G}(s) = -V_{0G} \exp\left(-\frac{s^2}{a_G^2}\right)$$

where $s$ denotes the pion–$\alpha$ relative separation. We recall that the Gaussian form has previously been considered to successfully represent the nucleon–$\alpha$ interaction [11, 29]. So, in this study we propose this form for the pion–$\alpha$ interaction but with slightly larger range. For the sake of comparison, another representation of the pion–$\alpha$ interaction is proposed in a Yukawa form as

$$V_{\pi-a}^{Y}(s) = -V_{0Y} \exp\left(-\frac{s}{a_Y}\right).$$

The values of the depths ($V_{0G}, V_{0Y}$) and ranges ($a_G, a_Y$) parameters are discussed in the next section. We denote the semi-microscopic potentials derived from expression (1) using the pion–$\alpha$ interactions (6) and (7) as $V_{\pi-a}^{G}$ and $V_{\pi-a}^{Y}$, respectively. These potentials are considered to represent the real parts of the pion–nucleus optical potentials.

In the folding model analysis, the folding procedure is usually directed to extract microscopic or semi-microscopic real parts of the optical potentials. For the imaginary parts, the folding model usually assumes [21] a parameterized form either with the WS shape or the surface peaked derivative of the WS form. In the present study, we adopt the conventional phenomenological local WS form as

$$W(R) = -W_0 \left[1 + \exp\left((R - r_i A_T^{1/3})/a_{i}\right)\right]^{-1}$$

where $W_0$ is the imaginary depth, $r_i$ and $a_i$ are the radius and diffuseness parameters, respectively, and $A_T$ is the target mass number. Then the total pion–nucleus potential $U(R)$ is expressed as

$$U(R) = V_C(R) + V_{\pi-a}^{G}(R) + iW(R),$$

where $V_C(R)$ is the Coulomb potential, which is considered here due to a uniformly charged sphere of radius $R_C = r_C A_T^{1/3}$ fm, with $r_C = 1.2$ fm.

2. Theoretical formalism

Considering the $\alpha$-cluster structure of the target nucleus, one may formulate the $\pi^\pm$-nucleus structure interaction in the framework of the SF model as [10, 11]

$$V_{SF}(R) = \int \rho_{CT}(r) V_{\pi-a}\left(|\vec{R} - \vec{r}|\right) \, d\vec{r},$$

(1)

where $\vec{R}$ denotes the projectile–target relative position vector and $\vec{r}$ is the position vector of an $\alpha$-cluster inside the target. The $\alpha$-cluster density distribution in the target ($^{12}\text{C}$ and $^{16}\text{O}$) nuclei $\rho_{CT}$ is expressed in the harmonic oscillator shape as

$$\rho_{CT}(r) = \rho_{OC} \left(1 + \gamma r^2\right) \exp\left(-\xi r^2\right).$$

(2)

This form can be extracted from the relation [10, 11]

$$\rho_m(r) = \int \rho_{CT}(\vec{r}) \rho_a\left(|\vec{r} - \vec{r}|\right) \, d\vec{r},$$

(3)

where $\rho_m$ and $\rho_a$ are the nuclear matter distributions of target nucleus and $\alpha$-particle, respectively, defined as

$$\rho_m(r) = \rho_{0m} \left(1 + w r^2\right) \exp\left(-\beta r^2\right),$$

(4)

$$\rho_a(r) = \rho_{0a} \exp\left(-\lambda r^2\right).$$

(5)

with

$$\xi = \frac{\beta \lambda}{\beta + \lambda} \quad \text{and} \quad \gamma = \frac{1}{4(\beta + \lambda)}$$

The parameters $\rho_{0m}, \rho_{0a}, w, \beta, \lambda$ used in the present calculations are given in our previous studies [10, 11].

In the recent studies [27, 28] a phenomenological WS form was used to express the pion- and kaon-$\alpha$ interactions, $V_{\pi-a}$ and $V_{K-a}$, to analyze the $\pi^\pm$ and $K^\pm$ elastic and inelastic scattering from $^3\text{Li}$ and $^{12}\text{C}$ in the framework of the Watanabe model. In the present study, we adopt two versions of the attractive $V_{\pi-a}$ potential to be used in the SF expression (1).
analyses of the used to determine collective properties of nuclei. For instance, potential. In addition, studying inelastic scattering data can be for other reaction channels, such as inelastic channels, since optical model potentials extracted from elastic scattering 

\[ \text{Table 1. Kinematic factors employed in a nonrelativistic Schrödinger equation in the present work with pion kinetic laboratory energy } E_{\text{lab}}. \]

\[
\begin{array}{c|c|c|c}
\text{ } & \text{ } & \text{ } \\
\text{E}_{\text{lab}} (\text{MeV}) & \text{E}_{\text{lab eff}} (\text{MeV}) & M_\pi (\text{u}) \\
\hline
\pi^- {\text{He}} & 110 & 89.30 & 0.25679 \\
& 180 & 137.18 & 0.32205 \\
& 260 & 189.54 & 0.39422 \\
\hline
\pi^- {\text{C}} & 100 & 80.23 & 0.25375 \\
& 120 & 93.75 & 0.27431 \\
& 150 & 113.30 & 0.30501 \\
& 180 & 132.20 & 0.33558 \\
& 200 & 144.53 & 0.35855 \\
& 230 & 162.73 & 0.38615 \\
& 260 & 180.66 & 0.41629 \\
& 280 & 192.49 & 0.43631 \\
& 400 & 262.35 & 0.55506 \\
& 486 & 311.85 & 0.63877 \\
& 500 & 319.89 & 0.65229 \\
& 584 & 368.18 & 0.73280 \\
& 663 & 413.69 & 0.80758 \\
& 766 & 473.31 & 0.90374 \\
\hline
\pi^- {\text{O}} & 114 & 89.41 & 0.26916 \\
& 162 & 120.34 & 0.31885 \\
& 240 & 167.64 & 0.39896 \\
\end{array}
\]

However, it has been long recommended [30] that HI optical model potentials extracted from elastic scattering should be tested by their ability to reproduce cross sections for other reaction channels, such as inelastic channels, since the form factors may probe different regions of the nuclear potential. In addition, studying inelastic scattering data can be used to determine collective properties of nuclei. For instance, analyses of the first 2+ state of even-even nuclei are expected to provide quadrupole deformation parameters (QDPs), β2, which could be determined either from electromagnetic measurements [31], or using hadronic probes, where a deformed potential (DP) is used to describe the inelastic scattering data [32].

The transition potential has a radial dependence as

\[ V_t(R) = -\delta_1 \frac{dV(R)}{dR}, \]  

(10)

where the deformation length δ1 determines the strength of the interaction and V(R) is the complex optical potential determined by the measured elastic scattering. The shape of Vt is independent of the multipolarity l of the transition. The DP (10) is often justified by arguing that the potential V(R) follows the shape of the density distribution when the latter is deformed. However, a more direct and consistent application of this view is to generate both optical and transition potentials by folding an effective NN interaction over the ground-state density of the projectile and the deformed density of the target [32]. The transition density may be obtained microscopically from the nuclear-structure calculation (such as one using the random-phase approximation), or macroscopically by deforming the ground state density distribution [32–34]. We adopt here the latter prescription where the transition density is obtained as

\[ \rho_t(r) = -\delta_1 \frac{d\rho_{\text{CT}}(r)}{dr}, \]

(11)

where \( \rho_{\text{CT}}(r) \) is the a-cluster density distribution as defined in equation (2). The deformation length δ1 is determined to fit the corresponding inelastic scattering data for the transitions to the 2+, 0+, 3− and 1+ states in 12C.

The imaginary transition potential is obtained from the derivative of the imaginary central potential as

\[ W_{t}(R) = -\delta_{1}^{\text{imag}} W_{0} \frac{d}{dR} \left[ 1 + \exp \left( R - r_{1} \gamma_{1}^{(2)} / a_{1} \right) \right]^{1} \]

(12)

where δ1 imag is the imaginary deformation length.

3. Procedure

The present work is devoted to investigating the \( \pi^- \)-nucleus elastic and inelastic scattering through a broad range of energies, 100–766 MeV. The motivation here is to test, for the first time, two versions of the effective pion–α interaction to construct a new pion-nucleus local optical potential in the framework of the SF approach as expressed in equation (1).

The nonrelativistic optical model computer codes DWUCK4 [35] and HIOPTIM-94 [36] are used to calculate the angular distributions of the elastic scattering differential cross section in the framework of the zero-range distorted wave Born approximation (DWBA). A reduced mass μ is considered as

\[ \mu = \frac{M_{\pi} m_{\pi}}{M_{\pi} + m_{\pi}} \]

(13)

where mπ is the target mass and the effective pion mass is 

\[ M_{\pi} = m_{\pi} + \gamma_{\pi} \]

where \( \gamma_{\pi} \) is defined as [3]

\[ \gamma_{\pi} = \left( y + \gamma_{L} \right) / \left( 1 + y^{2} + 2 y \gamma_{L} \right)^{1/2}, \quad y = m_{\pi} / m_{\pi} \]

\[ \gamma_{L} = 1 + \left( E_{\text{lab}} / m_{\pi} c^{2} \right). \]

(14)

where Elab is the pion bombarding energy in the laboratory system and mπ is the pion mass, \( m_{\pi} c^{2} = 139.6 \text{MeV} \). The center of mass kinetic energy \( E_{\text{cm}} \) is given as \( E_{\text{cm}} = (\hbar k)^{2} / 2 \mu \). Here \( \hbar k \) is the center of mass momentum of the incident pion. The effective bombarding energy \( E_{\text{lab eff}} \) is \( E_{\text{lab eff}} = M_{\pi} E_{\text{lab}} / \mu \). This will produce the appropriate k value in the form

\[ k = \left( m_{\pi} c / \hbar \right) \left( \gamma_{\pi}^{2} - 1 \right)^{1/2} = 4.72056 m_{\pi} \left( \gamma_{\pi}^{2} - 1 \right)^{1/2} \text{fm}^{-1} \]

(15)

where \( m_{\pi} = 0.1499 \text{ atomic mass unit} \). The resulting kinematic values \( M_{\pi} \) and \( E_{\text{lab}}^{\text{eff}} \) for the cases studied here are calculated according to the above equations and listed in table 1.
The routine searches are usually [20, 37] carried out considering an average value of 10% for all experimental data errors of the considered data to minimize the $\chi^2$ value which is defined as

$$\chi^2 = \frac{1}{N_D} \sum_{i=1}^{N_D} \left( \frac{\sigma_{th}(\theta_k) - \sigma_{exp}(\theta_k)}{\Delta \sigma_{exp}(\theta_k)} \right)^2,$$

where $N_D$ is the number of differential cross-section data points, $\sigma_{th}(\theta_k)$ is the calculated cross section at angle $\theta_k$ in the c.m. system, $\sigma_{exp}(\theta_k)$ and $\Delta \sigma_{exp}(\theta_k)$ are the corresponding experimental cross section and its relative uncertainty, respectively. All parameters used in the calculations of the real SF potentials, $V_{SF}^G$ and $V_{SF}^Y$, from equation (1) are held constant during the search except the depths and together with the imaginary WS parameters ($W_{ri}$, $a_i$) are freely adjusted in order to fit the data. The values of the range parameters used in the calculations are proposed to be roughly 2.13 fm and 0.8 fm, respectively.

Inelastic calculations are carried out using the DWUCK4 code based upon DWBA by introducing the transition real and imaginary potentials as mentioned in the previous section to investigate the transition to the $2^+, 0^+, 3^-$, and $1^+$ excited states in $^{12}$C. Searches are performed only on the real and imaginary deformation lengths in order to obtain the best fits with the corresponding inelastic scattering data. No changes are taken into account in the best fit optical potential parameters obtained from the elastic scattering analysis.

### 4. Results and discussion

#### 4.1. $\pi^- - ^4$He scattering

In order to confirm our choice of the proposed pion–α interactions, we check first their ability to produce a successful description of the pion-alpha elastic scattering at several bombarding pion energies. The real part of $^4$He potential calculated by expressions (6) or (7) together with the imaginary part of WS with parameters given in table 2 are used to calculate the differential cross sections for $\pi^-$ scattering from $^4$He at energies of 110, 180 and 260 MeV. The depths $V_{G\alpha}$ and $V_{Y\alpha}$ are fixed at 30 MeV for the three considered energies. The resulting angular distributions for the elastic scattering are shown in figure 1, and are compared with the experimental data [38]. The resulting kinematic parameter values for the cases studied here are calculated according to the equations given in section 3 and are collected in table 1. The two forms of potentials used in the present work give almost similar results especially at forward angles. The only significant difference between the Gaussian and Yukawa results for elastic scattering was found at 260 MeV at backward angles. Elastic scattering results using either form of potentials near the resonance energy were found to be nearly indistinguishable, as shown in figure 1. It can be seen from table 2 that at each energy the same imaginary potential parameters are used for both Gaussian and Yukawa interactions. Further, it is evident that the imaginary parameters decrease with increasing energy. The extracted total cross sections are in excellent agreement with the corresponding

<table>
<thead>
<tr>
<th>$E_{lab}$ (MeV)</th>
<th>$W_G$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\sigma_T$ (mb)</th>
<th>$\sigma_T$ (mb)</th>
<th>$\sigma_T$ (mb)</th>
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</thead>
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<td>0.6275</td>
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<td>0.3550</td>
<td>333.62</td>
<td>340.12</td>
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<tr>
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<td>0.4822</td>
<td>238.55</td>
<td>249.39</td>
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</tbody>
</table>

**Figure 1.** Data for differential elastic cross section scattering at 110, 180 and 260 MeV $\pi^-$ from $^4$He [38], are compared with the results of the real Gaussian (solid curves) and Yukawa (dashed curves) SF potentials with the imaginary WS parameters of table 2.
measured values, particularly when using the Gaussian interaction. This significant success of the pion–α elastic scattering descriptions motivated us to introduce these adopted pion-alpha interactions (6) and (7) into the SF calculations to construct semi-microscopic pion-nucleus real optical potentials.

4.2. \( \pi^-\)nucleus scattering

The SF model based upon the α-cluster structure is employed to generate local optical potentials in order to analyze the \( \pi^-\) scattering on \(^{12}\)C and \(^{16}\)O at energies of 114, 162 and 240 MeV. The best parameters deduced from matching our results with data are listed in tables 3 and 4. We recall that, in order to extract the energy dependence of the strength of the interaction, only the depth of the \( \pi^-\alpha\) interaction is considered as a free parameter during the calculation to build up the real part of the \( \pi^-\)nucleus optical potential. The resulting energy dependence of \( V_G\) and \( V_Y\) are displayed in figure 2 for \(^{12}\)C, as an example. As can be seen from this figure, the extracted depths of both Gaussian and Yukawa interactions, are clearly energy dependent. For instance, it is noticeable that the depth of the Gaussian interaction \( V_{0G}\) grows from 12.41 MeV to 27.34 MeV as energy increases from 100 MeV to 200 MeV and has a maximum value (32.5 MeV) at 270 MeV. Then, at laboratory bombarding energies \( E_{lab} > 300\) MeV the interaction depth decays rapidly. Similar behavior is also found for the depth of the Yukawa interaction \( V_{0Y}\) i.e., both depths reveal very similar behaviors. The \( V_{0Y}\), however, is apparently larger than \( V_{0G}\) over all the considered energy range. This

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**Table 3.** Best fit optical potential parameters for \( \pi^-\)\(^{12}\)C elastic scattering for the Gaussian and Yukawa SF potentials.

<table>
<thead>
<tr>
<th>( E_{lab} ) (MeV)</th>
<th>( W_0 ) (MeV)</th>
<th>( r_i ) (fm)</th>
<th>( a_i ) (fm)</th>
<th>( \chi^2 )</th>
</tr>
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<td>G</td>
<td>Y</td>
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<td>65.29</td>
<td>1.1203</td>
<td>0.9981</td>
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</table>

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**Table 4.** Best fit optical potential parameters for \( \pi^+\)\(^{16}\)O elastic scattering for the Gaussian and Yukawa SF potentials. Also shown, total cross sections for \( \pi^+\) scattering on \(^{16}\)O calculated in the present work compared to those of [44]. The depth parameters used for the Gaussian and Yukawa interactions are 23.16, 15.33, 13.75 and 24.45, 17.64, 15.15 MeV, respectively.

<table>
<thead>
<tr>
<th>( E_{lab} ) (MeV)</th>
<th>( W_0 ) (MeV)</th>
<th>( r_i ) (fm)</th>
<th>( a_i ) (fm)</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
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<td>( W_0 ) (MeV)</td>
<td>( r_i ) (fm)</td>
<td>( a_i ) (fm)</td>
<td>( \chi^2 )</td>
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<tr>
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<td>50.55</td>
<td>1.7490</td>
<td>0.4050</td>
<td>690.29</td>
</tr>
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**Figure 2.** The energy dependence of the best fit depths of \( \pi^-\alpha\) Gaussian and Yukawa interactions used in the SF potential calculations. \( E_{lab} \) is the pion bombarding energy in the laboratory system.
behavior of energy dependence is correlated to the resonance energy region, where a maximum strength is found near the resonance energy (∼200 MeV). As energy increases higher than the resonance region, the potential strength apparently decays to weaker values.

All considered sets of pion–12C elastic scattering data in the present work (except at 100 MeV) have been, previously, analyzed by Hong and Kim [5] in the framework of a phenomenological representation of the local optical potentials using the conventional WS form with six free parameters. So, it may be worthwhile to compare our derived SF potentials with those obtained in [5].

As an example for π−12C, we present in figure 3 a comparison between the real part of the present SF and previous WS [5] potentials that successfully reproduced the elastic scattering data at the incident pion energy $E_{\text{lab}} = 200$ MeV. Around this energy the pion–nucleus scattering is dominated by strong absorption arising from the resonance character of the pion–nucleon amplitude. Several remarks can be extracted from figure 3. First, in the interior region ($R < 2.0$ fm), the Yukawa potential is substantially shallower than the Gaussian one. However, both potentials reveal similar values at the surface region ($R \geq 5.0$ fm). In addition, it is evident that the WS potential starts very deep with respect to the derived SF potentials, near the origin, then sharply decays to become identical to the Yukawa one (and shallower than the Gaussian potential) at $R \geq 3.0$ fm. So, this figure confirms how far the predicted scattering cross sections of incident pions at 200 MeV are insensitive to details of the pion–12C optical potential in the interior region. Similar behavior is expected for the other energies regarding the corresponding depths as shown in figure 2. It is also worth mentioning that the derived Gaussian and Yukawa SF potentials have energy-independent root mean square (rms) radii of values 3.24 and 2.73 fm, respectively.

The angular distributions of $\pi^\pm$12C elastic scattering differential cross section predicted by the constructed SF potentials are shown in figures 4–6 in comparison with the corresponding measured data [39–42] at all considered energies. For the sake of comparison, we also present the predictions obtained in [5] using phenomenological WS potentials. Solid and dashed lines represent results of the Gaussian and Yukawa SF potentials, respectively, while dotted lines represent the WS [5] ones. In general, a good description of the 14 sets of data is obtained using both SF potentials. The quality of the present fits is apparently better than that of [5], particularly at backward angles although the latter have more flexibility (six free parameters) to match the data. At lower energies, the Gaussian potentials seem to produce better results than the Yukawa ones. One may also note from table 3 that, for Yukawa potentials, the fitting criterion parameter $\chi^2$ has, relatively, better (smaller) values at higher energies than those of the Gaussian potentials and vice versa at lower energies. Furthermore, as evident from figures 4–6, the present results show that the oscillatory structure of the angular distributions is qualitatively well reproduced using the derived SF potentials. The present results confirm the success of the α-cluster model as an effective tool in the folding formalism. Consequently, the present results reveal additional evidence for the ability of SF approximation to generate a realistic representation of the pion–nucleus interaction as well as nucleus-nucleus ones.

Recall that, in the present study, our investigations are confined to the attractive real parts of the $\pi^\pm$12C optical potentials. We did not consider repulsive potentials at energies just above the Δ-resonance energy as done in some previous studies [5, 6, 43]. It was found [5] that, through this energy region, cross sections calculated using attractive potentials were indistinguishable from those calculated by repulsive potentials. Our efforts are concentrated here on checking the feasibility of introducing an effective $\pi$–α interaction expressed either in a Gaussian or Yukawa form in the SF calculations, and how far the extracted pion–12C potentials can successfully reproduce the measured elastic scattering cross sections. We recall that a rough choice of the range parameters ($a_G$ and $a_Y$) is considered for this task. Of course, fine adjustments of these parameters are insistently recommended and may be investigated in further studies in the near future.

For the sake of confirmation we introduce the proposed $\pi$–α interactions (6) and (7) in the folding calculation to extract the SF potentials to investigate the pion–16O elastic scattering cross sections at three energies. The method employed here for deriving SF potential can be used for 16O nucleus (or similar nuclei) at appropriate energies. We are able to obtain a good fit to the data [44] for the elastic scattering of 114, 162 and 240 MeV $\pi^+$ from 16O with calculations based upon the two forms of potentials together with the imaginary part of Woods–Saxon with parameters given in table 4. The Gaussian local potential calculations are much more comprehensive than the Yukawa one. These calculations show negligible differences between the predictions of the differential cross sections by the two potentials used here, as shown in figure 7.
Another quantity, which measures the validity of the adopted effective interactions, is the reaction (absorption) cross section, $\sigma_R$. Our scattering calculations based on the two SF potentials are also used to predict the reaction cross sections $\sigma_R$ of negative pions scattering from $^{12}\text{C}$ at the considered energies. A comparison between our extracted values for negative pions and those of [45–47] is shown in figure 8. It can be seen from this figure that there is a very good agreement between the present results and previous ones [45–47]. This indicates that the imaginary part of the optical
potential used here, which is strongly correlated to $\sigma_R$, is well predicted. There is a strong absorption of pions at the resonance energy region confirmed by higher values of $\sigma_R$ while a weaker absorption is noticed at lower and higher incident pion energies. This behavior is somehow different from that found for composite projectiles [48, 49], where there is a strong absorption at low incident energies with higher values of $\sigma_R$. 

Table 5. Derived quantities for $\pi^\pm$ scattering from $^{12}\text{C}$ using parameters in Table 3. For each energy the first line is for Gaussian potential while the second line is for the Yukawa one. (See text for details.)

<table>
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<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$\sigma_{1/2}$ (fm)</th>
<th>$\sigma_{\text{t}}$ (MeV)</th>
<th>$\sigma_{\text{r}}$ (MeV)</th>
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<td>1.64</td>
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<td>616.15</td>
<td>18.55</td>
<td>2.34</td>
</tr>
<tr>
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<td>529.24</td>
<td>20.48</td>
<td>2.27</td>
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Figure 7. Data for differential elastic cross section scattering at 110–260 MeV $\pi^+$ from $^{16}\text{O}$ [44], are compared with the results of the real Gaussian (solid curves) and Yukawa (dashed curves) SF potentials with the imaginary WS parameters of Table 4.

Figure 8. Total and reaction cross sections for scattering of negative pions from $^{12}\text{C}$ are shown. The present calculations are compared with experimental data from [43, 47] for the total cross sections and from [45–47] for the reaction cross sections.

Figure 9. The energy dependence of the deduced imaginary volume integral $J_I$ from the phenomenological WS supplemented with the real Gaussian and Yukawa SF potentials.
which simply falls off as the energy increases and stays around a certain value for each intermediate or high energy reaction as transparency effect.

Since the pion mean free path $\lambda$ is proportional to the inverse of the total $\pi$-nucleon cross sections, i.e. $\lambda \propto \frac{1}{\sigma_{\pi A}}$ [50], it is easy to deduce from figure 8 that $\lambda$ for pions is shorter in the resonance region than those in higher and lower regions. This confirms that the pions are strongly absorbed at the surface of target nuclei. From figure 8 it can also be seen that for pions scattering from $^{12}$C nucleus both calculated total

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**Figure 10.** Data for differential inelastic cross section scattering at 100–280 MeV $\pi^-$ exciting the 4.44 MeV $2^+$ state of $^{12}$C. The experimental data are taken from [42] and compared with the results of the Gaussian (solid curves) and Yukawa (dashed curves) SF potentials.

**Figure 11.** As in figure 10, but for $\pi^-$ exciting the 9.64 MeV $3^-$ state of $^{12}$C.
and reaction cross sections, $\sigma_T$ and $\sigma_R$, have similar energy dependence shapes. Results reveal that the two SF potentials produce resonance at $\sim$180 MeV. Regarding the measured values of $\sigma_T$ for $\pi^-\alpha$ scattering listed in table 2 and those displayed in figure 8 for $\pi^-{^{12}}C$ we find that both systems reveal resonance at 180 MeV, similar to that noticed for pion-nucleon in free space. Our SF potentials successfully predicted this resonance consistency.

Table 5 shows the predictions of the DWUCK4 computer code used in the present work for the partial wave angular momentum $L_{1/2}$ corresponding to the strong absorption radius $D$. Extracted values for the imaginary $J_I$ volume integral per nucleon of the target nucleus, the Im $V(D)$ nuclear potential at the distance $D$ and the rms radius $<r^2>_\text{Im}$ of the imaginary nuclear potential are also shown in table 5. It is well known that at the strong absorption radius $D$ the incident particle has a 50% probability of being absorbed by the target nucleus. The strong absorption radius $D$ is related to $L_{1/2}$ according to

$$kD = \left[ L_{1/2}(L_{1/2} + 1) + \eta^2 \right]^{1/2} + \eta,$$

where $\eta$, $\mu$, and $k$ are, respectively, the Sommerfeld parameter, reduced mass and center of mass wave number. From table 5, it is clear that, for both SF potentials, $L_{1/2}$ increases as the energy of the incident pion increases, particularly at energies above the resonance region. This behavior is consistent with that previously found for negative pion scattering from $^{12}$C at several energies [5, 37]. The values of the strong absorption radius $D$ at energies above the resonance region are relatively smaller than those at energies below the resonance region. Furthermore, it is obvious that at higher energies, the values of the rms radius of the imaginary potential $<r^2>_\text{Im}$ are comparable to the corresponding $D$ values and at the same time both $<r^2>_\text{Im}$ and $D$ values are relatively smaller than those of the $<r^2>_\text{Re}$ mentioned in the previous subsection. Also, it is seen that the volume integral per nucleon of the target nucleus of the absorptive potential, $J_I$, increases with the increase of energy of the incident particle up to the resonance energy. This behavior may be attributed to the more expected inelastic channels to be opened at the resonance energy [51]. The shape of the energy dependence of $J_I$ is shown in figure 9. We note that, for both Gaussian and Yukawa potentials, the change of $J_I$ with energy resembles the energy dependence of the corresponding real depths shown in figure 2. It is obvious from figure 9 that, all over the considered angular range, the value of $J_I$ for the Gaussian potential is larger than that for the Yukawa one although the real depths of the Yukawa potential in figure 2 are higher than those of Gaussian potential. We remind the reader that we do not need to list the corresponding values of the real volume integral $J_R$ in table 5, since $J_R$ is linearly proportional to the depth of the SF potential, where $J_R = 13.49 (1.61) \times 10^{19}$ MeV fm$^3$ for the Gaussian (Yukawa) potential, and the values of the depths are already displayed in figure 2.

**Table 6.** Deformation lengths (in fm) for the 4.44 MeV $2^+$, 7.6 MeV $0^+$, 9.64 MeV $3^-$ and 12.7 MeV $1^+$ states of $^{12}$C compared to those predicted by others.

<table>
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<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>State</th>
<th>Present $\delta$</th>
<th>Others</th>
<th>Ref.</th>
<th>Present $\delta$</th>
<th>Others</th>
<th>Ref.</th>
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<tr>
<td></td>
<td></td>
<td>$\delta_{\text{real}}$</td>
<td>$\delta_{\text{mag}}$</td>
<td>$\delta$</td>
<td>$\delta_{\text{real}}$</td>
<td>$\delta_{\text{mag}}$</td>
<td>$\delta$</td>
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Figure 12. As in figure 10, but for $\pi^n$ at 100 MeV exciting the 7.6 MeV $0^+$, 12.7 MeV $1^+$, 4.44 MeV $2^+$, and 9.64 MeV $3^-$ states of $^{12}$C. The experimental data are taken from [40].
4.3. Inelastic scattering

Inelastic scattering is another important aspect of the interactions of pions with nuclei. In a phenomenological approach, it is expected that both elastic and inelastic scattering will be described in the same framework. This consistency may provide more information on the pion-nucleus potential and is essential before reliable nuclear structure information can be expected. With the zero-range DWBA, we compute inelastic scattering to the collective $2^+, 0^+, 3^-$ and $1^+$ states of the target $^{12}$C using the DWUCK4 code. For the present deformed potentials, the values of the real and imaginary deformation lengths $\tilde{\sigma}^{\text{real}}$ and $\tilde{\sigma}^{\text{imag}}$ are adjusted to obtain a reasonable description of 19 sets of inelastic scattering data through the energy range 100–280 MeV. No change is carried out on the potential parameters obtained from the elastic scattering calculations. The results are shown in figures 10–12. The Coulomb excitation is found to be unimportant for the inelastic scattering cases considered here. The deformation lengths obtained from the best fits between calculated and measured inelastic cross sections are listed in table 6 and compared with those theoretically obtained from previous studies [52–54]. From this table it is evident that the obtained deformation lengths of $^{12}$C for all considered excited states are quite consistent with those previously found using different techniques of calculations.

Angular distributions of the inelastic scattering cross sections of $\pi^-$ by $^{12}$C ($2^+; 4.4$ MeV) and ($3^-; 9.64$ MeV) states at 100–280 MeV pion kinetic energies are presented in figures 10 and 11, while angular correlations for the inelastic scattering of $\pi^+$ by $^{12}$C ($0^+; 7.6$ MeV), ($1^+; 12.7$ MeV), ($2^+; 4.4$ MeV), and ($3^-; 9.64$ MeV) states at 100 MeV are shown in figure 12. The experimental angular distributions displayed in these figures are generally smooth, with well defined minima. The angular distributions of the $\pi^-$–$^{12}$C inelastic differential scattering cross section using both derived SF potentials are similar in all cases, apart from small shifts in the locations of some minima that are observed. These shifts are comparable to those seen in the elastic scattering calculations. From table 6, it can be seen that values of the deformation lengths determined here using the two SF potentials forms are very similar to those obtained from the phenomenological WS potentials of [5]. It is also clear from this table that the deformation lengths extracted from the SF real potentials are substantially larger than the corresponding ones extracted from the phenomenological WS imaginary potentials in all cases under consideration. So, the extracted inelastic scattering results evidently prove the ability of the deformed SF potentials to successfully describe the shapes and magnitudes of the measured inelastic scattering data [40, 42].

5. Conclusion

In the present study we generated semi-microscopic local optical potentials based upon the SF formalism and the $\alpha$-cluster model in order to analyze pion-nucleus scattering through a broad range of energies, 100–766 MeV. Two phenomenological forms of the pion-alpha (Gaussian and Yukawa) interaction have been proposed in order to construct the SF pion–$^{12}$C and $^{16}$O real potentials. The imaginary potentials were phenomenologically represented in the conventional three--parameter WS form. The proposed pion-alpha interactions have been used first to successfully produce the pion-alpha elastic scattering cross sections. Then, both interactions were introduced as ingredients in the folding calculations. The extracted real SF potentials produced successful descriptions of 36 sets of angular distributions of the $\pi^-–^{12}$C and $^{16}$O elastic and inelastic differential cross section. Both adopted interactions revealed similar energy dependence behaviors characterized by larger strengths around the $\Delta^-$ resonance region. The obtained predictions of angular distributions of elastic scattering data are found to be better than, or at least very similar to, those previously obtained using complex phenomenological WS optical potentials [5]. In addition, the extracted values of reaction and total cross sections for the three considered pion-nucleus reactions are in good agreement with the corresponding experimental data, as well as those reported in previous studies using different calculation techniques. Furthermore, the values of the deformation lengths extracted from all considered excited states of $^{12}$C are quite consistent with those of previous estimations.

In summary, one may conclude from the present results that the proposed Gaussian and Yukawa forms are able to produce realistic representations of the effective $\pi$-$\alpha$ interaction required for the SF formulations. We propose that both interactions can be useful for the analysis of elastic, and consequently inelastic, scattering of pions on light HI based upon the $\alpha$-cluster structure of the target nuclei in the framework of the SF approach. We do not intend to claim that the ranges and strengths just quoted are precisely determined. Of course, they are not, they just provide a good starting point for further investigations. A further benefit of the present study is that we deduce additional evidence for the successful performance of the $\alpha$-cluster model in analyses of reactions involving light heavy ions in the framework of the SF approach, as very recently achieved for the double folding one [55–58].

Acknowledgments

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