A Model Of New Polishing Process to Fused Deposition Modeling Parts

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Abstract:
The present paper presents a heat transfer model to a new polishing process that smoothens the surface of products manufactured by fused deposition modeling (FDM) technology. FDM is one of the most common additive manufacturing techniques which, unfortunately, results in very rough surfaces when compared to similar technologies. This high roughness results from the stepped surface (stair like surface) caused by the technology nature of depositing several 2D layers to form the final 3D product. To smooth the surface, it is heated up till melting which trigger surface tension force to smooth the surface. To achieve surface melting, the surface is exposed to a localized hot air jet with certain temperature and velocity from a moving nozzle with appropriate translational velocity; this introduces three main process parameters: air jet temperature, air jet velocity and air nozzle translational velocity over the product surface. Two analytical heat transfer models were derived using different process parameters and proved to be in agreement with each other. Also, a part of the obtained experimental results verifies model results. Moreover, the effect of entrained air on the heated jet was considered in the model. It can be concluded that we have an analytical model fits the experimental one and represents the modeled process.

Keywords: FDM, modeling of polishing process, hot air, impinging jet, Surface Roughness

List of symbols:

- \( d_i \): inner diameter of annular nozzle
- \( L_{W} \): The plane wall half thickness
- \( L \): The distance in z direction between the nozzle face and the impinged surface
- \( L_{C} \): Characteristic length, for pipe flow it equal D of the pipe
- \( d_o \): The outer diameter of annular nozzle
- \( R \): Dimensionless radial distance, \( R = \frac{r}{d_i} \), r is the radial distance
- \( A_{1.8} \): The circular area over dimensionless R=1.8
- \( \alpha \): Thermal diffusivity, \( \alpha = \frac{k}{\rho \cdot C_p} \)
- \( \nu \): Kinematic viscosity of air
- \( \eta^* \): The dimensionless jet temperature at parts surface with the definition of Hollworth and Wilson
- \( \dot{m}_{\text{sur}} \): The fused or melted mass from the part surface
- \( \overline{\dot{h}}_{av,R} \): Average convective heat transfer value over area of \( R=1.8 \)
- \( \overline{\dot{h}}_{av} \): Average convective heat transfer value over circular area of dimensionless radius R. \( \overline{\dot{h}}_{av,R} = \frac{\dot{h}_{av,L_{C}}}{k_{air}} \)
- \( \overline{\dot{h}}_{av,L_{C}} \): The local Nusselt number, the dimensionless convective heat transfer
- \( \overline{\dot{h}_{av}} \): The value of average Nusselt number mentioned by [6].
- \( \overline{\dot{h}_{av,L_{C}}} \): The average Nusselt over a circular area of dimensionless radius R. \( \overline{\dot{h}_{av,L_{C}}} = \frac{\dot{h}_{av,L_{C}}}{k_{air}} \)
- \( k \): Thermal diffusivity, \( \alpha = \frac{k}{\rho \cdot C_p} \)
1. **Introduction:**

FDM is the most common additive manufacturing technologies owing to its low cost simple fabrication technique and ability to manufacture complicated 3D closed contour parts. The technology deals with wide range of polymer base material which found its way in industrial application and may also non-polymer materials under research phase like ceramic or metal. Its manufacturing process is extruding a semi molten material from a nozzle carried in a 3-axis automated machine. FDM machine manufactures parts by extruding a semi-molten filament material through a robotically controlled nozzle in machine head; the nozzle heats and extrudes the material while the machine head is moving to deposit the layers of the part, see **fig.1**.

![Figure 1: Description of FDM technology](image_url)

**Figure 1:** description of FDM technology (a) the machine[17] (b) the nozzle within manufacturing (c) the resulting surface roughness

Owing to this layer wise manufacturing technique, a stair like surface will result and this effect is the source of the elevated roughness which additive manufacturing characterized with, see **fig.1b**. Unfortunately, FDM has high product roughness between additive technologies [1] and a post-processing (machining) Method are generally employed for smoothing FDM parts.

The investigated post-processing technique employs mainly a heat transfer process where a hot air jet impinges the surface of FDM parts in order to melt locally the surface (the outer layer) - **Figure 2** shows a schematic and experimental result of this polishing process. Melting triggers surface tension force which flattens the surface and finally leaves it smoother after naturally cooled down in free air. In a microscopic scale, the surface stairs will collapse to fill or heel the gaps in between these melted stairs.
The technique is similar to laser polishing which generally employed to smooth hard alloys with a significant roughness reduction. The difference between the two techniques is that laser heats surfaces up by radiation while hot air employs convection. Also, the polished materials for laser are metal or ceramic alloys while it is polymer material for hot air (material of FDM parts). The process requires impinging a jet of hot with proper temperature and velocity. In order to provide continuous machining, the hot air nozzle moves with a proper translational velocity over the surface that is lower enough to cause melting. Similar to laser polishing, it was also found that much lower nozzle velocity cause overheating and surface deterioration. In other hand, high nozzle velocity causes insufficient heating and no smoothening to the surface and hence, proper tuning to the process parameter is required. Another important point is that the process response depends to several parameters and a prediction to process behavior is required when a change in process parameters or part material is introduced.

Also air polishing will provide lower machining cost and safer processing condition for possible manual applications. for impinging jet application, The process generally can deliver a surface density of thermal power transfer rate up to $1\text{Gw/m}^2$ [2] like in cooling of turbine blade making the process applicable in this process. The hot air heating process has its potentials over using laser process as the process has low initial cost. Also the most appropriate laser used in polymer based materials as our technology is CO$_2$ laser owing to its appropriate absorbity (wavelength $\approx$10600 μm) and so less reflectivity, but it has low efficiency (a common value 15%) in his high voltage power supply which may make hot air processing more safe and probable efficient solution -as will be seen- especially if used with amateur application.

Fortunately, FDM uses low melting material (polymer materials) making the method applicable. Furthermore, beside surface tension force, additional forces related to

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**Fig. 2:** (a) 3D general description of the polishing process (b) 2D description of finishing phenomena at line shown- a sintering phenomenon
the developed process help moving (flattening) the fused stair tips to the corresponding valleys around which are stagnation pressure/force and shear flow force that help flattening the surface.

Impinging of hot air jet is a forced convection heat transfer problem. Its main input output parameter is jet exit velocity and the corresponding convective heat transfer coefficient respectively. The jet impinges the FDM part surface with a certain temperature as an input. With a certain convective heat transfer coefficient (h) and at certain temperature, the surface takes an optimum amount of time (or optimum nozzle translation velocity) to be heated to melting. When surface melting is reached, the effect of different forces stated appears naturally and redistribution to its topography happens leaving the surface smoother as shown in see fig.2a.

2. Modeling of the finishing process:

Figure 1c shows the surface profile of the parts manufactured by FDM process. At the surface of the product, the stairs appear physically which can be recognized by eye in real world. The polishing process intended to melt the shell of the FDM product surface. Melting will achieved by exposing the surface to a hot air jet. The jet is impinged to the surface with proper temperature and velocity and for a certain appropriate exposure time (certain nozzle velocity). Figure 3 shows a description of model parameters.

The process model starts with a heated jet exit from a nozzle opening and then, passes through a stagnant fresh air for certain distance till impinges the processed surface for a certain amount of time (or velocity). This time (or the corresponding velocity) is the time required to melt a certain amount of material from the surface. In this study, the investigated parameters will be jet temperature, jet velocity, and processing time/velocity. This process sequence is investigated in the three following sections: heat transfer of impinging jet, entrainment effect on heated jets, and a convenient heat transfer model of the polishing process. The following two sections will investigate the equations describing these stages beginning from determining the convective heat transfer of the jet and the jet temperature at part surface. The result of these two sections will be feed to the third one which is a 2-D transient – forced convection heat transfer model. The third section will and the required processing time/velocity based on two transient –forced convection heat transfer models.

2.1 Heat transfer from the impinging jets:

Impinging jet is an advanced topic in heat transfer and less amount of work presented to determining the dimensionless heat transfer coefficient (Nu number). The process contains different process parameters including type of flow, nozzle
shape, fluid type, surrounding conditions, nozzle configurations and hard to be determined in generalized equation form. This section handles the circular impinging jet (CIJ) while Figure 3a process parameters for CIJ. CIJ is simply generated from a nozzle with circular cross section (pipe type). CIJ is the common type between impinging jets owing to its simple nozzle section and high resulting Nusselt number over small area. On the negative side, almost all experimental studies presented in CIJ are using non laminar flow application or related to certain fluid or conditions which is not our case [3]–[5]. Turbulence flow results to high stagnation pressure which may deteriorate the polished surface upon melting.; Hence, only laminar flow

Figure 3: (a) the parameters of circular impinging jet (b) Nu number for annular jets at different Re number, (regenerated with permission from Elsevier publisher with license number 3699481203810)

will be employed in the study. Greene et al [3] (chapter 2) presents a good review for CIJ and provides analytical for laminar flow (based on laminar flow theories), but it employs either uniform surface temperature or uniform heat flux which is not our case.

Chattopadhyay [6] presents the most related study to our model conditions. The study presents a numerical results for CIJ compared with annular impinging jet in the resulting Nusselt number for an investigated domain of R=5d, the study reported Nu values for from 250 up to 1000. Figure 3b shows the data obtained from Chattopadhyay for AIJ and CIJ. It can be noted that the average Nusselt values given in the study calculated by \( \text{Nu}_{\text{aver}} = \frac{1}{R} \int_0^R \text{Nu} \, dr \). To obtain the average Nu value over circular area of radius \( R=r/d_i \), we calculated \( \text{Nu}_R = \frac{2}{R^2} \int_0^R \text{Nu} \, r \, dr \). Table 1 contains the \( \text{Nu}_R \) values for different Re number integrated numerically over the area of interest of \( R=1.8 \); this area is about the double of the stagnation region which is about at \( R=0.9 \) at figure 3a; also, this area was considered as the heat-affected zone
while Nu values at radius larger than R=1.8 will be small and usefulness for melting and will be omitted. Table 1 also shows the Chattopadhyay model conditions are jet height "h"-from impinged surface - is two times the characteristic length (Lc =2d). Also, the ambient temperature is equal to jet temperature (cooling process) and/or no entrainment effect. Also uniform velocity gradient in the output from the nozzle was assumed. Our model will follow up the

**Table1: results of Nusselt number with Reynolds number before and after perform numerical integration to the result given in [6] using our operating parameter.**

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>The study result</th>
<th>the numerically calculated results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Circular Nuaverr</td>
<td>Difference ratio</td>
</tr>
<tr>
<td>1000</td>
<td>10.16</td>
<td>19.6%</td>
</tr>
<tr>
<td>500</td>
<td>6.88</td>
<td>21.4%</td>
</tr>
<tr>
<td>250</td>
<td>4.49</td>
<td>25.6%</td>
</tr>
<tr>
<td>Best fit curve</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

study conditions assuming it prevails except the two conditions that T = T∞ (or cooling process) and the other condition that the effect of "entrainment effect" is neglected – as investigated in the following.

2.2 Entainment Effect:

When an impinging jet crosses its nozzle opening to a stagnant medium, an amount of fresh air entrained with the perimeter of the jet. Figure 4 presents a schematic to the phenomenon. This mix between the ambient fresh air and the high jet temperature causes decrease in jet temperature when reached to part surface for both axial and radial flow directions. When the jet cuts longer distance in the ambient, the entrainment effect becomes more significant and so, the nozzle exit temperature is no longer the actual processing temperature.

Jambunathan et al [7] presents a review in entrainment effect. A dimensionless jet parameter - named effectiveness- is studied to indicate jet actual temperature at any axial and radial position. These studies state that effectiveness is independent to Reynolds number and jet temperature. Also, the effectiveness is near unity (approximate no change in jet temperature) at small apparent distance between nozzle and target surface – length to diameter (L/d_{noz}) <4. For larger apparent distances L/d_{noz}, the effectiveness in stagnation region become significant for larger L/d_{noz}. While radial flow region, it is still independent on all values L/d_{noz}. Moreover, the value of resulting Nu number is independent to the temperature difference
between jet and ambient air. Goldstein [8] presents a correlation of \( \eta^* \) by Hollworth and Wilson definition as:

\[
\eta^* = \frac{T_{aw} - T_\infty}{T_{jet} - T_\infty} \approx 0.38 + 0.6e^{-0.01\left(\frac{L}{d_i}\right)^2-0.1\left(r/d_i\right)^{2.5}}
\]

(2)

And hence, the average effectiveness value over the studied area of \( R=1.8 \) will be:

\[
\bar{\eta}_{av} = \frac{T_{aw} - T_\infty}{T_{jet} - T_\infty} = \frac{2}{\left(r/d_i\right)} \int_0^{r/d_i=0.18} \eta^* \left(\frac{r}{d_i}\right) \cdot d \left(\frac{r}{d_i}\right) = 0.879 \quad \text{where} \quad \frac{L}{d_i} = 2, \ r/d_i = 1.8
\]

(3)

The value of \( \bar{\eta}_{av} \) in eq.3 was used to obtain the average jet temperature over the polished surface with respect jet exit temperature \( T_{jet} \). This temperature will be feed to the model at section 2.3.

2.3 heat transfer models of the Polishing process:

This section will presents the employed heat transfer models to use to model the process. As previously state, laser Polishing is the closest process to our developed one; it is more commonly applied in metal based additive manufacturing technology [9], [10]. Figure 4 illustrates the similarities between laser and air polishing processes.

![Figure 4: comparison between the developed polishing process in (a) and laser polishing process in (b).](image_url)

This similarity helps developing the model with guidance to similar studies in laser polishing process. Bordatchev et al [11] represent a review to the process including the modeling studies. He concluded that almost all were based on some of the classical theories developed for heat transfer by conduction. Ramose et al [12] presents an analytical model laser polishing process to selective laser sintering processed parts (SLS). The SLS technology is one of additive manufacturing technology with similar characteristics to the used one. For the analytical model, the studies present an energy balance equation for laser beam where a part of energy
input in the surface will melt and superheat a certain mass from surface and the
other part of energy will heat up another amount of surface material but below
melting temperature. In our process, the surface will pass with the same sequence
with the aid of impingement of hot air instead of laser. Sections 2.1 and 2.2 presents
the equivalent Nu number (the convective heat transfer) and air jet temperature at
the surface respectively. This section will determine the third parameter which is the
processing time (or the corresponding velocity) to melt the surface. This processing
time is required to melt the outer microscopic layer which contains the staircase (the
rough topography). This processing time is the required to heat up the surface to
melting \( t_{\text{heating}} \) and the time required for phase change of the surface to liquid
\( t_{\text{phase change}} \) as in \( \text{eq. 4}. \)

\[
t_{\text{processing}} = t_{\text{heating}} + t_{\text{phase change}}
\]  

(4)

The term \( t_{\text{phase change}} \) depends on thickness of the molten layer to successfully melt
the staircase in this layer. This thickness is about the measured surface roughness
value \( R_a \) which is extremely small -in order of 1E-6 meter- and hence, the term
\( t_{\text{phase change}} \) is neglected. Therefore, the term \( t_{\text{processing}} \) is reduced to \( t_{\text{heating}} \). This
time \( (t_{\text{heating}}) \) can be obtained by solving this transient conduction problem. The
best model fits this circumferences is a semi-infinite solid model exposed to
temperature of \( T_\infty = T_{\text{jet}}, h_{av} \). this model fits polymer or insulating materials like
earth. the book [13] (chapter5, equation 5.63) presents the model equation as
follows:

\[
\frac{T(x,t_{\text{heating}}) - T_i}{T_\infty - T_i} = \text{erfc} \left( \frac{x}{2\sqrt{\kappa t_{\text{heating}}}} \right) - \left[ \exp \left( \frac{\bar{h}_{av} x}{k} + \frac{\bar{h}_{av}^2 \kappa}{k^2} t_{\text{heating}} \right) \right] \text{erfc} \left( \frac{x}{2\sqrt{\kappa t_{\text{heating}}}} + \frac{\bar{h}_{av} \kappa}{k} t_{\text{heating}} \right)
\]  

(5)

The value of \( x \) is considered zero and \( \kappa \) and \( k \) is the thermal diffusivity and thermal
conductivity of the FDM part material respectively. The model valid within treated
part thickness more than the penetration depth \( \delta_p \) at which
\( \frac{T_{\delta_p} - T_{\text{surface}}}{T_{\text{initial}} - T_{\text{surface}}} = 0.9 \)
which is reasonable thickness for polymer materials that has much low thermal
diffusivity is the case for FDM material.

The data output from the model will also validated with data results from
considering the surface as one dimensional transient conduction problem in plane
wall of thickness 2 \( L_w \) where \( \delta_p > L_w \). The model equation is:

\[
\frac{T(x,t_{\text{heating}}) - T_0}{T_{\text{initial}} - T_\infty} = \sum_{n=1}^\infty C_n \exp \left( -\frac{\pi^2 R_F}{L_w^2} \right) \cos \left( \xi_n \frac{x}{L_w} \right) \text{ where } C_n = \frac{4 \sin \xi_n}{2 \xi_n + \sin (2 \xi_n)}
\]  

(6)

\( \text{Eq.7} \) [13] (equations 5.63 and 5.42a) is the infinity Fourier series where \( F_0 = \frac{L_w}{\alpha t_{\text{heating}}} \).
The thermo-physical properties used in the model (along figure 6, 7) are averaged and presented in table 2 for poly(lactic acid) (PLA). Also, air properties are averaged over its operating temperature. The properties are taken from [14], [15].

Table2: Properties of PLA used. Values are averaged in operating temperature range 35°C to 175°C

<table>
<thead>
<tr>
<th>Item</th>
<th>Density ($\bar{\rho}$) Kg/m³</th>
<th>Specific heat ($C_p$) J/Kg.K</th>
<th>Thermal conductivity ($K$) w/m.K</th>
<th>heat of fusion ($\Delta H_m$) j/kg</th>
<th>Melting temperature ($T_m$) °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLA</td>
<td>1192</td>
<td>2227</td>
<td>0.205</td>
<td>45000</td>
<td>175</td>
</tr>
</tbody>
</table>

3. Results and discussion:

This section will present the study results divided into two subsections. The first one section presents the obtained results after operating different model equation based on table 2. The second subsection will present the model verification with experimental results based on table 3 after implementing the exact material properties and the actual experiment conditions.

3.1 modeling results:

Using the equations previously presented from 1 through 6, the relation between different process parameters is obtained by the two proposed models. The two

![Figure 5: the relation between processing time and velocity at constant temperature for circular impinging jet models were matched with maximum error of 2.6% and average of 1.1% and presented in Figures from 5 to 6.](image)
Fig 5 shows the model results with different processing parameters. The polished area has a dimensionless radius \( R = 1.8 \) \((r=1.8d)\) and hence, the process velocity can be computed by dividing the diameter of area of interest by the obtained time. Figure 5 shows the relation between jet exit velocity and nozzle velocity for different jet exit temperature. Figure 6 shows the relation between jet exit temperature and nozzle velocity for different jet exit velocity. It appears from both figures that jet temperature has more effect than jet velocity in accelerate the process. A worth noting is that the model was feed with much lower jet velocity which results to much lower nozzle velocity as shown in figures 5, 6. These results help us to predict and work at higher jet velocity and hence, higher nozzle velocity as will be presented in the experiment section.

![circular nozzle diagram](image)

**Figure 6:** the relation between processing time and temperature at constant velocity for circular impinging jet and

3.2 experimental verification:

This section presents the experimental data obtained for the investigated polishing process. This section was held after modeling studies where more accurate and related properties were obtained as shown in table 3; The site of Ultimachine cites the common PLA type of PLA4043D-Natureworks [16]. The peak melting temperature is 145-165 °C which with average of 153 °C as in table 3.
Table 3: Properties of PLA used. Values are averaged in operating temperature range 35°C to 153°C

<table>
<thead>
<tr>
<th>Item</th>
<th>Density (ρ) Kg/m³</th>
<th>Specific heat (Cₚ) J/Kg.K</th>
<th>Thermal conductivity (K) w/m.K</th>
<th>heat of fusion ∆H_m J/kg</th>
<th>Melting temperature T_m °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLA</td>
<td>1256</td>
<td>2227¹</td>
<td>0.165</td>
<td>45000</td>
<td>153</td>
</tr>
</tbody>
</table>

Figure 7a shows the experimental test rig while figure 7b show an experimentally polished surface with magnification to the boundary region between the polished and the unpolished region.

Figure 7b shows the experimentally polished surface and demonstrate the concept shown in figure 2b. A ninety one different tested surface under different testing parameters was held. Figure 8 shows the comparison between the model data represented by continuous line and the experimental one represented by discrete points. The model data is obtained corresponds to properties at table 3. Also the experimental processing conditions are: jet velocity from 14 to 28.5 m/sec. and jet temperature from 215 to 265°C. The experimental data corresponds to the highest achieved surface roughness reductions (smoothest surface) which introduce...
sufficient melting to the surface as derived in the model. This roughness reduction is from 85% to 66% which from initial range of surface roughness from 7 to 8 μm.

![Graph](image)

**Figure 8:** The model and the experimental results for the relations between nozzle translation velocity and (a) jet exit velocity (b) jet exit velocity

The deviation of model from experimental results is considered acceptable compared with similar studied published in polishing. The difference between the
model and the experimental results may be owing to the deviation between the exact material properties and the averaged one. This material properties are four with other approximations at Nu value. Also the exit temperature was measured 1cm apparent from the exit opening which may contribute in this deviation. A worth noting is that the model can fit the experiment better than it be by a bit change in the thermo physical properties that at table3 while we committed to the thermo physical properties obtained from [16] while it is known that PLA properties dependent on the molecular weight and stereo-chemical makeup of its backbone [14].

4. Conclusion:

A modeling to a new polishing process is presented to improve the surface roughness of FDM parts. The developed polishing process replaces the laser as heating source by an impinging hot air jet to melt the outer surface layer to improves the surface roughness. An analytical model was developed by studying impinging jet heat transfer, entrainment in impinging jet, and 2-D transient conduction models to relate the main process parameter: jet temperature, jet velocity, and nozzle translational velocity. The model data was validated using two heat transfer model giving the same results and verification experimentally with a good agreement. Furthermore, the model can predict the effect of other process parameters including material properties. It is found that jet temperature can accelerate the polishing process more than jet velocity. Base on the developed model, we can predict the behavior of different process parameters.

Reference:


2004.


