Cylindrical Damped Solitary Propagation in Superthermal Plasmas


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Abstract—Wave properties of damped solitons in a collisional unmagnetized four components dusty fluid plasma system contains superthermal distributed electrons, mobile ions and negative-positive dusty grains have been examined. To study dissipative DIA mode properties, a reductive perturbation (RP) analysis is used under convenient geometrical coordinate transformation, three dimension damped Kadomtsev–Petviashvili equation 3-CDKP in cylindrical coordinates is obtained. Effects of collisional parameters on damped soliton pulse structures are studied. More specifically, the coordinate impact of axial, radial and polar with the time on solitary propagation are examined. This investigation may be viable in plasmas of Earth’s mesospher.

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1. INTRODUCTION

Collisional complex dusty plasmas in space and earth magnetosphere is the plasma fluid containing ions, electrons, neutrals, and liquid or solid dust micro grains [1–4]. The charged micro grains in plasma dynamics changes the plasma characteristics aside from introduction of new wave phenomena types i.e. dust-acoustic (DA) and dust ion acoustic (DIA) waves [5–7]. The existence of (DIA) solitons has been confirmed by an experimental laboratory investigation [8, 9]. Popel et al. studied DIA waves in plasmas by using an approximation that neglecting electrons- ions scattering and absorption by micro grains [10]. Experimentally, it was noted that Dust laboratory plasmas contain a number of neutrals and the dusty-neutral collision affect the wave properties in plasma [11, 12]. The effect of dust-neutrals collisions on the wave behavior in plasma were taken into account [13]. Popel et al. reported that the anomalous wave dissipation causes the existence of new types of shock waves that important for description star formation and shocks in supernova [14]. Some articles studied the ion recombination frequency on micro particles, frequency of momentum-transfer for ion-particle (ion neutral) collisions [15–18]. Moslem investigated the magnetic field effect on the damping rate of collisional dusty plasma [15]. El-Labany et al. examine the variable charges and electron trapping on the propagating DIAs characteristics in collisional dusty plasmas [17]. Many discussions introduced the negative charged dusty plasma applications in space plasma [19, 20]. Furthermore, other articles, inspected both the negative and positive charged micro dust grains in laboratoried and space plasmas [21–23]. On the other hand, Maxwellian distribution is one of the most distributions used in fluid plasma dynamics to describe the thermal equilibrium states. When non-equilibrium states exist, this distribution may be unusual for plasma description [24–26]. More specifically, several authors introduced the non-equilibrium kappa distribution for electron, positron (ion) in the plasmas of astrophysical space [27–34]. However, a lot of these studies are concerning to the unbounded coordinates geometry. This is not true for space and laboratory plasma. So, the nonplanar form of cylindrical geometry is taken into account [35, 36]. Theoretical investigations in non-planar geometry for wave existence in dusty plasmas have been discussed [37–40]. In this work we aim to examine the damping wave behavior for the nonplanar (cylindrical) damped three-dimensional DIAWs in collisional dusty plasma model contains superthermal distributed electrons, mobile ions and negative-positive dust micro grains. Organization of this paper is in the form: in Section 2 we introduced model equations. In Section 3 the derivation of CDKP equation is present. Its damped solution is obtained in Section 4. Section 5 is devoted for some discussions.
2. SYSTEM OF EQUATIONS

A four components dusty collisional plasma system contains mixture of superthermal distributed electrons, mobile ions and negative-positive dust grains. For mobile components Three-dimensional continuity equations are given by

$\frac{\partial n_i}{\partial t} + \nabla (n_i u_i) = -v_{ri} n_i + v_i n_{ex}$, \hspace{1cm} (1a)

$\frac{\partial n_e}{\partial t} + \nabla (n_e u_e) = 0$, \hspace{1cm} (1b)

$\frac{\partial n_p}{\partial t} + \nabla (n_p u_p) = 0$. \hspace{1cm} (1c)

The corresponding momentum equations are,

$\frac{\partial}{\partial t} (n_i u_i) + \nabla (n_i u_i^2) + n_i \nabla \phi = -v_{ri} n_i$, \hspace{1cm} (2a)

$\left( \frac{\partial}{\partial t} + u_j \nabla \right) u_n - \mu \nabla \phi = 0$, \hspace{1cm} (2b)

$\left( \frac{\partial}{\partial t} + u_p \frac{\partial}{\partial x} \right) u_p + \alpha \nabla \phi = \nu_p \mu_p$. \hspace{1cm} (2c)

Where the two terms $v_{ri} n_i$ and $v_{ri} n_e$ are approximately given by $v_{ri} (n_i = n_{e0})$ and $v_{ri} (n_e - n_{e0})$ [14] under assumption of $\tau_{\Theta} (charging time scale) = 0$. These equations are coupled by the Poisson equation

$\nabla^2 \phi = \delta_n n_n - \delta_p n_p - \frac{1}{\kappa} \phi$, \hspace{1cm} (3)

In the above equations $n_j (j = i, n, p, e)$ are the perturbed number densities and $n_{i0}, n_{n0}, n_{p0}$ and $n_{e0}$ are the related equilibrium values. Also $u_j (j = i, n, p)$ are the ion, negative and positive dusty plasma velocities, respectively, normalized by the ion sound velocity $(K_B T_j / m_j)^{1/2}$. $\phi$ is a potential and normalized by $\left( K_B T_e / e \right)$, $t$ and space coordinate are normalized by the plasma frequency $\omega_p \cdot K_B T_e / \left( 4 \pi e^2 n_{e0} \right)^1/2$ and electron Debye length $\lambda_e = (K_B T_e / 4 \pi e^2 n_{e0})^{1/2}$, respectively. $v_{ri}$ is the recombination frequency of ions on dust particles, $v_i$ is the ionization frequency of plasma, $\bar{v}$ is a frequency for loss in ion momentum due to the recombination on dust particles and collisions between ions and grains of dust, $v_{pn}$ is a frequency for loss in positive dust momentum due to negative-positive dust collisions. The collision frequencies $v_{iex}, v_i, v_e$ and $v_{pn}$ are normalized by $\omega_p \cdot \mu = Z_n m_e / e$, $\alpha = Z_n m_e / e$. Here, $K_B$ and $T_e$ Boltzmann constant and temperature of electron, $e$ the electronic charge, $m_j (j = i, n, p)$ denote ion-negative and positive dust masses respectively. The superthermal electrons density $n_e$ is given by

$n_e = \left( 1 - \frac{\phi}{\kappa - 3/2} \right)^{\kappa + 3/2}$. \hspace{1cm} (4)

where $\kappa$ is the spectral index parameter. Expanding $n_e$ in $\phi$, we get

$\nabla^2 \phi = \delta_n n_n - \delta_p n_p + 1 + \beta_1 \phi + \beta_2 \phi^2$, \hspace{1cm} (5)

where

$\beta_1 = (\kappa - 1/2) / (\kappa - 3/2)$, \hspace{1cm} $\beta_2 = (\kappa + 1/2) / (2(\kappa - 3/2))$. \hspace{1cm} (6)

From the change neutrality condition, we have

$\delta_n + 1 = \delta_p + \delta_e$. \hspace{1cm} (7)

with $\delta_i = n_{i0} / n_{i0}$, $\delta_e = Z_n n_{e0} / n_{e0}$, $\delta_p = Z_p n_{p0} / n_{e0}$, where $Z_n$ and $Z_p$ are the charge numbers of negative and positive dust, respectively. $\kappa$ is a real parameter measuring deviation from the Maxwellian equilibrium (which is recovered for $\kappa \rightarrow \infty$).

3. Nonlinear calculations

To study DIA waves properties, a reductive perturbation (RP) analysis is used [41]. We introduce the new independent variables [42, 43]:

$R = \epsilon^{1/2} (r - \lambda t)$, \hspace{1cm} $\Theta = \epsilon^{-1/2} \phi$, \hspace{1cm} $Z = \epsilon z$, \hspace{1cm} and \hspace{1cm} $T = \epsilon^{3/2} t$, \hspace{1cm} (8)

where $\epsilon$ is a small parameter measures the degree of perturbation and the $\lambda$ is velocity of wave propagation.

We have assumed that $v_{ri} \sim \epsilon^{1/2} v_{ri0}$, $v_i \sim \epsilon^{1/2} v_i0$, $\bar{v} \sim \epsilon^{1/2} \bar{v}_0$, and $v_{pn} \sim \epsilon^{1/2} v_{pn0}$. All variables in the model are expanded in the powers of $\epsilon$ as

$n_j = 1 + \epsilon u_j^{(1)} + \epsilon^2 u_j^{(2)} + \ldots$, \hspace{1cm} (9)

$u_j = \epsilon u_j^{(1)} + \epsilon^2 u_j^{(2)} + \ldots$, \hspace{1cm} (10a)

$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \ldots$, \hspace{1cm} (10b)

$\Omega = \epsilon \Omega^{(1)} + \epsilon^2 \Omega^{(2)} + \ldots$, \hspace{1cm} (10a)

$\nu_p = \epsilon \nu_p^{(1)} + \epsilon^2 \nu_p^{(2)} + \ldots$, \hspace{1cm} (10b)

$w_j = \epsilon w_j^{(1)} + \epsilon^2 w_j^{(2)} + \ldots$, \hspace{1cm} (10b)

$w_j = \epsilon w_j^{(1)} + \epsilon^2 w_j^{(2)} + \ldots$, \hspace{1cm} (10b)

$w_j = \epsilon w_j^{(1)} + \epsilon^2 w_j^{(2)} + \ldots$, \hspace{1cm} (10b)

where $u_j, v_j$ and $w_j$ are the ion, negative and positive dust velocities in $R, \Theta, Z$. Directions. Using Eqs. (8), (9) in Eqs. (1), (2) and (5), first-order in $\epsilon$ for ion are

$n_i^{(1)} = \frac{1}{\lambda^2} \phi^{(1)}$, \hspace{1cm} $u_i^{(1)} = \frac{1}{\lambda} \phi^{(1)}$, \hspace{1cm} (10a)

$\frac{\partial n_i^{(1)}}{\partial R} = \frac{1}{T \lambda^2} \frac{\partial \phi^{(1)}}{\partial \phi}$, \hspace{1cm} (10b)

$\frac{\partial w_i^{(1)}}{\partial R} = \frac{1}{\lambda} \frac{\partial \phi^{(1)}}{\partial Z}$, \hspace{1cm} (10b)

$\frac{\partial w_i^{(1)}}{\partial R} = \frac{1}{\lambda} \frac{\partial \phi^{(1)}}{\partial Z}$, \hspace{1cm} (10b)
whereas for negative dust are given by

\[
\eta_n^{(i)} = -\frac{\mu}{\lambda^2} \phi^{(i)}, \quad \eta_n^{(i)} = -\frac{\mu}{\lambda} \phi^{(i)}, \tag{11a}
\]

\[
\frac{\partial \eta_n^{(i)}}{\partial R} = -\frac{\mu}{T\lambda^2} \frac{\partial \phi^{(i)}}{\partial \Theta}, \quad \frac{\partial \eta_n^{(i)}}{\partial R} = -\frac{\mu}{\lambda} \frac{\partial \phi^{(i)}}{\partial Z}, \tag{11b}
\]

and for positive dust are given by

\[
\eta_p^{(i)} = \frac{\alpha}{\lambda^2} \phi^{(i)}, \quad \eta_p^{(i)} = \frac{\alpha}{\lambda} \phi^{(i)}, \tag{12a}
\]

\[
\frac{\partial \eta_p^{(i)}}{\partial R} = \frac{\alpha}{T\lambda^2} \frac{\partial \phi^{(i)}}{\partial \Theta}, \quad \frac{\partial \eta_p^{(i)}}{\partial R} = \frac{\alpha}{\lambda} \frac{\partial \phi^{(i)}}{\partial Z}. \tag{12b}
\]

Poisson equation leads to the compatibility condition:

\[
\lambda^2 = \frac{\delta_i + \delta_n \mu + \delta_p \alpha}{\beta_i}. \tag{13}
\]

At second order in \( \varepsilon \), the nonsecularity condition for second order quantities leads to 3-CDKP equation in the form

\[
\frac{\partial}{\partial T} \left( \frac{\partial \phi^{(i)}}{\partial T} + \frac{\phi^{(i)}}{2T} + A \frac{\partial \phi^{(i)}}{\partial R} + B \frac{\partial^2 \phi^{(i)}}{\partial R^2} + C \phi^{(i)} \right) + \frac{1}{2\lambda^2} \frac{\partial^2 \phi^{(i)}}{\partial \Theta^2} + \frac{\lambda^2}{2} \frac{\partial^2 \phi^{(i)}}{\partial Z^2} = 0, \tag{14}
\]

where

\[
A = \frac{3}{2\beta_i \lambda^2} (\delta_i - \delta_n \mu^2 + \delta_p \alpha^2 - 2\beta_i \lambda^4 / 3), \tag{15a}
\]

\[
B = \frac{\lambda}{2\beta_i}, \tag{15b}
\]

\[
C = \frac{1}{2\beta_i \lambda^2} (\delta_i (\nu_{re} + \nu_0 - \nu_{jo} \beta_i \lambda^2) + \delta_p \alpha \nu_{pl0}). \tag{15c}
\]

Equation (14) is a nonlinear evolution of \( \phi^{(i)} \) with coefficients of nonlinear and dispersion terms \( A, B \) and dissipation coefficient \( C \). For neglecting the \( Z \) and \( \Theta \) dependence, Eq. (14) reduced to DKdV equation.

### 3.1. 3D-CKP Solution

The damped 3-CKP Eq. (14) is not solvable exact in analytic form. However, for a weak dissipation effects (damping due to collisions) and by introducing the transformation

\[
\chi = R_L \beta + Z \sqrt{1-L^2} - \tau \left( \frac{1}{2} \phi^2 \lambda L + U_0 \right), \tag{16}
\]

one can obtain an approximate form of analytical solution for Eq. (14) \([44–46]\):

\[
\phi(\chi, \tau) = \phi(\tau) \text{sech}^2 \left( \frac{a}{b} \frac{\phi(\tau)}{2\sqrt{3}} \right), \tag{17}
\]

where the amplitude and width of damped soliton \( \phi(\tau) \) and \( L(\tau) \) are given by:

\[
\phi(\tau) = \phi_0(0) e^{-\frac{4C^2}{3}}, \tag{18}
\]

\[
L(\tau) = \frac{2\sqrt{3} e^{\frac{4C^2}{3}} \sqrt{a \phi_0(0)} e^{-\frac{4C^2}{3}}}{b \phi_0(0)}, \tag{19}
\]

where \( \phi_0(0) \) is the amplitude of the single localized soliton in absence of the damping coefficient \( C = 0 \) and \( U_0 \) satisfied the condition \( U_0 = \frac{\lambda(1 - \beta)}{2L_L} \).

### 4. RESULTS AND DISCUSSION

To carry out this study in a four components collisional fluid plasma model, by applying reductive perturbation theory, introduce by Taniuti and Wei \([41]\), this plasma model leads to a 3D-CDKP Eq. (14). Now we discuss the effects of some fluid plasma parameters on the nature of the damped solitary waves, using mesospheric parameters \([47, 48]\). For adequate result, the time-evolutions of the dust ion acoustic solitons are plotted in Fig. 1, which confirms the existence of weakly dissipative soliton structures in the four components dusty plasma system. The credence of both the damped solitary amplitude and width of the DIAs
on the spectral index parameter $\kappa$ and time $\tau$ are graphically depicted in Figs. 2, 3. Figures 2 and 3 show that, amplitude and width of the damped solitary profile increase with $\kappa$. However, the damped soliton amplitude decreases with time $\tau$ and the damped soliton width increases with $\tau$. Hence one of importance and necessary aim in this paper is to investigate the collisional effects in plasma wave properties such as: ionization frequency ($\nu_{i}$), ion recombination frequency on dust particles ($\nu_{re0}$), the frequency due to recombination on dust particles and Coulomb elastic collisions between ions and dust ($\nu_{0}$) and the frequency due to the elastic collisions between dust particles ($\nu_{p0}$). The effects of these frequencies are studied for the solitonic damped amplitude and growing width $\phi_{1}(\tau)$ and $L(\tau)$ in Figs. 4–7. It is noted that both $\nu_{0}$ and $\nu_{re0}$ decreases damped solitary amplitude and increases its width as shown in Figs. 4, 5. Furthermore, in Figs. 6, 7 the variations of $\phi_{1}(\tau)$ and $L(\tau)$ with $\nu_{0}$ and $\nu_{p0}$ have been plotted. By increasing $\nu_{0}$ values the amplitude $\phi_{1}(\tau)$ increases and width $L(\tau)$ decreases as advised in Fig. 6. In contrast, Fig. 7 indicated that the increase of $\nu_{p0}$ values decreases $\phi_{1}(\tau)$ and increases width $L(\tau)$. On the other hand, Figs. 8, 9 depicted the damped solitary profile $\phi$ in Eq. (17) with the geometric variables $\Theta$, $R$, $Z$, and $\tau$. It is found that, soliton profile tends to swerve to positively radial axis and damped with time as displayed in Fig. 8. Finally, DIA damped soliton is shown in Fig. 8 with $\tau$ and $\Theta$ axis for variable values of $R$. It was reported that sharp deviation and damping causes an intense geometric distortion in soliton form as plotted.
Fig. 6. (Color online) Graph of $\phi_0(\tau)$ versus $\nu_{pn0}$ for $\nu_{i0}$ for $\alpha = 0.002$, $\mu = 0.005$, $L_r = 0.5$, $\delta_n = 1.3$, $\delta_p = 1.5$, $\nu_{re0} = 0.5$, $\bar{\nu}_0 = 0.2$ and $\kappa = 2$, $\tau = 1$.

Fig. 7. (Color online) Graph of $L(\tau)$ versus $\nu_{pn0}$ for $\nu_{i0}$ for $\alpha = 0.002$, $\mu = 0.005$, $L_r = 0.5$, $\delta_n = 1.3$, $\delta_p = 1.5$, $\nu_{re0} = 0.5$, $\bar{\nu}_0 = 0.2$ and $\kappa = 2$, $\tau = 1$.

Fig. 8. (Color online) 3D profile $\phi(\chi, \tau)$ against $R$ and $\Theta$ for $Z = 0.5$ and for $L_r = 0.5$, $\alpha = 0.002$, $\mu = 0.005$, $\delta_n = 1.3$, $\delta_p = 1.5$, $\nu_{re0} = 0.5$, $\bar{\nu}_0 = 0.3$, $\nu_{i0} = 0.2$, $\nu_{pn0} = 0.3$ and $\kappa = 2$ for (a) $\tau = 1$ and (b) variable $\tau$.

Fig. 9. (Color online) 3D profile $\phi(\chi, \tau)$ against $R$, $\Theta$ and $\tau$ for $Z = 0.5$ and for $L_r = 0.5$, $\alpha = 0.002$, $\mu = 0.005$, $\delta_n = 1.3$, $\delta_p = 1.5$, $\nu_{re0} = 0.5$, $\bar{\nu}_0 = 0.3$, $\nu_{i0} = 0.2$, $\nu_{pn0} = 0.3$, $\kappa = 2$ for (a) $R = 0.5$, (b) variable $R$. 
in Fig. 9. To summarize, we have examined the nonlinear damping structures of DIAIs in a multi component, unmagnetized, collisional dust plasmas having superthermal electrons, mobile cold ions, positive and negative grains in cylindrical geometry. The radial and angular dependence of the system physical quantities have been depicted. The effects of geometric, index and collisional plasma system parameters ($\kappa$, $\nu$, $\psi_{\text{col}}$, $\psi_{\text{e}}$, $\Theta$, $R$ and $Z$ and $\tau$) on the damped amplitude $\phi(t)$ and width $\lambda(t)$ have been inquiring numerically using mesoscopic parameters [47, 48]. We have shown graphically that these value parameters introduced a necessary function in figuration and brows of DIA damped solitons. Furthermore, the existence of geometric distortion regions of solitons are investigated. Also, our study on damped solitary form could be beneficial for better realization nonlinear damped wave features on a multicomponent fluid plasmas in laboratory, space and astrophysical mediums.

REFERENCES